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SEADYN MATHEMATICAL MODELS

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An Investigation Conducted by
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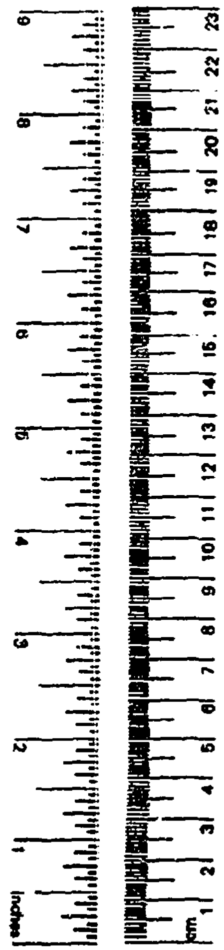
METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

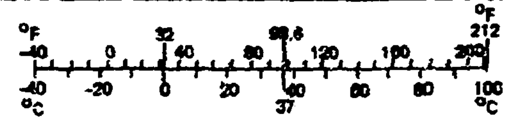
Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2,000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1,000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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This manual presents the theoretical background material for the SEADYN cable, truss, and mooring program. SEADYN uses the finite element method for modeling the cables, trusses, and mooring lines. Two elements are treated: the one-dimensional Simplex (truss) element and a bottom-limited catenary. Lumped parameter concepts are used in treating buoys, anchors, floats,

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etc. Rigid body models are used for ships, platforms, mooring buoys, etc. A cartesian 3-D geometric space is used throughout. Besides describing the element and body equations for submerged responses with large displacements, this manual discusses the various static and dynamic solution methods employed.

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CONTENTS

1.0 INTRODUCTION

The SEADYN computer program came into existence in 1974 as an extension of efforts to model underwater electronic structures at the Electronic Systems Division of the General Electric Company. The major features of the program were developed as part of the author's doctoral studies at Cornell University [1]. In 1975 the Naval Civil Engineering Laboratory (NCEL) began evaluating SEADYN and the Chesapeake Division of the Naval Facilities Engineering Command funded extensions to include moored vessel options. The output of that funding was the SEADYN/DSSM program and associated documentation [2-5]. Since 1977 the Naval Facilities Engineering Command through NCEL has provided continuous development and experimental verification of the program. This activity has led to extensive modifications of the program and the addition of new capabilities. These include:

1. A major reorganization of the program structure and the adoption of a free-field input format.

2. The addition of line payout/reel-in capabilities.

3. The addition of the strumming model of Skop, Griffin and Ramberg [6].

4. The development of the viscous relaxation solution for static analyses [7].

5. The addition of the bottom-limited catenary element.

6. The addition of material internal damping models.

7. The addition of the time sequenced static solutions option.

8. The development of a plotting post-processor [8].

9. The addition of a body impact model (developed for U. S. Coast Guard R&D Center).

The extent of these developments has prompted the production of a new set of documentation. Three volumes have been written: the present volume, a user's manual [9], and a programmer's reference manual [10]. The purpose of this manual is to summarize assumptions, equations, and numerical solution methods used in SEADYN.

Finite element techniques based on the stiffness method are employed in SEADYN. The hallmark of stiffness methods is the versatility of structural form allowed. Quite complicated structural arrangements, loads and boundary conditions are permitted. Unlike many special purpose cable programs, SEADYN makes no restriction on connection topology or geometric form. Multiple connected redundant systems and networks are modeled as readily as single line spans. Since the basic element is a straight line segment, SEADYN can be used to model truss structures as well as cable systems.

Specific physical characteristics which are important in cable and mooring systems lead to nonlinearities in the equations. Some of them are:

Geometric Nonlinearity - The system stiffness depends on preloading and the deflections of the system.

Postion Dependent Loading - Loads delivered to the system depend on the position and orientation of the system and they change as the system moves. This is typical of fluid induced loadings for drag and inertia loads (added mass).

Nonlinear Loading - Loads which depend not only on position, but are a nonlinear function of the system state variables, e.g., fluid drag loads depend on the square of the relative velocity and Reynolds' number.

Postion Dependent Constraints - The system must remain within specified constraints, e.g., surface and bottom limits.

Nonlinear Materials - The load/strain relationship is dependent on the amount of strain.

Physical Changes in the Structure - The structure itself may be modified with time; snagging a body or paying out lines are in this category.

Each of these phenomena are treated by SEADYN. This manual outlines how this is done.

2.0 BASIC MODELING ASSUMPTIONS

The approach taken in the SEADYN computer program to model cable and mooring systems can be described as a discrete element approach. It can be considered as a combination of the finite element method and the lumped parameter method in which lines are modeled by the finite element method with bodies being lumped at the node points.

In its classical form, the finite element method seeks to represent continuous physical systems with a set of discrete or finite elements which are formulated by assuming the character of the element response in terms of a set of interpolating functions. In its usual form it is equivalent to a Galerkin form of the method of weighted residuals where the weighting functions are defined individually on each element. Viewed from another perspective, the finite element method is a form of the Rayleigh-Ritz method in which the trial functions are defined only on individual subregions (elements) of the system. The basic equations for cables, mooring lines and hawsers are obtained using a simple finite element in the form of a straight line. The element is assumed to be straight both before and after deformation of the system, but no restriction is placed on the amount of stretch and/or rotation the element is subjected to. It is further assumed that bending and torsional effects are negligible. In the case of bending this means that the bending stiffness of the cable has negligible influence on the global response of the system. Neglect of the torsional effects does not mean that twist is unimportant. It simply means that coupling between twist and extension is assumed to have little effect on the overall shape and response of the system. An obvious situation where this is an invalid assumption is in low tension conditions where a twist instability may result in kinking or hockling. In addition to the straight line element, a catenary model is provided for bottom interaction. This element neglects fluid loads and roughly approximates mass redistribution with the bottom interaction.

The only deformable components in the system are assumed to be the cable elements. The material is assumed to be hyperelastic, i.e., non-linear, time-independent with loading and unloading curves coincident. The frequency domain allows proportional damping effects while the transient dynamic model allows various forms of material damping.

Any component of the system which cannot be modeled as an individual line element or a set of line elements is assumed to be a rigid body. These rigid components are assumed to be lumped at a single point in the system. They may be assumed to have only a point effect or to act as a rigid connector for arbitrarily placed lines (e.g., a ship).

The system may be totally immersed in a fluid, suspended between two fluids (e.g., water and air) or fluid effects may be ignored. The treatment of fluid effects makes the fundamental assumption that the fluid and structure problems are uncoupled. This means that except for specific localized effects the overall fluid field characteristics are unaltered by the presence of the structure. Thus, such things as flow alteration due to structural movement and blockage effects are not dealt with. More specifics on the assumptions and limitations of the fluid interaction with the structural system are discussed in Section 3.5 and Reference 1.

The Lagrangian approach is taken in describing the motion of the system. In this approach all physical variables are expressed in terms of their values at an intitial reference state. It is possible to change the reference state by employing generalized coordinate transformations which account for distortions and rotations. Analytical procedures which begin from a reference state and never change that reference are called total Lagrangian. Updated Lagrangian is an obvious title for methods which periodically move or update the reference state. Either procedure can be used and the results obtained should be equivalent. In the developments which follow the configuration of a system (or an element) is designated by the capital letter C and a pre-superscript. The symbol ${}^{R}C$ means the reference configuration while ${}^{t}C$ means the configuration at some time, t. The definition of quantities like stress, strain and displacement usually involve two configurations. A pre-subscript is used to denote the reference configuration for such cases. Thus, ${}^{t}_{o}C$ means a quantity in ${}^{t}C$ measured relative to ${}^{o}C$.

Details of the finite element method applicable to cable systems are given in Reference 1. Only brief summaries of the results pertinent to the SEADYN program are given here.

3.0 SUMMARY OF GOVERNING EQUATIONS

3.1 Global Equation Forms

The general form of the equations of motion for an element can be written

$$ {}^{t}_{R}[M]\ {}^{t}_{R}\{\ddot{q}\} = {}^{t}\{f\} - {}^{t}\{g\} = {}^{t}\{R\} \qquad 3\text{-}1 $$

where

${}^{t}_{R}[M]$ is the element mass matrix

${}^{t}\{f\}$ represents the external nodal forces in ${}^{t}C$

${}^{t}\{g\}$ represents the nodal reactions in ${}^{t}C$

${}^{t}\{R\}$ is called the force residual

An incremental form of the motion equations can be written

$$ {}^{t}_{R}[M]\ \{\Delta\ddot{q}\} = \{\Delta f\} - {}^{t}_{R}[K_T]\ \{\Delta q\} - {}^{t}_{R}[C]\ \{\Delta\dot{q}\} \qquad 3\text{-}2 $$

where

$$ \{\Delta q\} = {}^{t+\Delta t}_{R}\{q\} - {}^{t}_{R}\{q\} \qquad 3\text{-}3 $$

and

${}^{t}_{R}[K_T]$ is called the tangent stiffness matrix

${}^{t}_{R}[C]$ is an incremental damping matrix

In many situations (e.g., fluid loading) the force is dependent on the deflection. In this case

$$ \{\Delta f\} = \frac{\partial\ {}^{t}\{f\}}{\partial\ t}\ \Delta t + \frac{\partial\ {}^{t}\{f\}}{\partial\ {}^{t}_{R}\{q\}}\ \{\Delta q\} \qquad 3\text{-}4 $$

$$ = \{\Delta\bar{f}\} + {}^{t}_{R}[K_R]\ \{\Delta q\} $$

The incremental motion equations are then written

$$ {}^{t}_{R}[M]\ \{\Delta\ddot{q}\} + {}^{t}_{R}[C]\ \{\Delta\dot{q}\} + {}^{t}_{R}[\bar{K}_T]\ \{\Delta q\} = \{\Delta\bar{f}\} \qquad 3\text{-}5 $$

where

$$ {}^{t}_{R}[\bar{K}_T] = {}^{t}_{R}[K_T] - {}^{t}_{R}[K_R] \qquad 3\text{-}6 $$

Equation (3-2) or (3-6) can be used to model small displacement response about a steady deformed configuration, or it can be used in nonlinear dynamics by recalculating the stiffness matrix at each step. It should be noted that both equations neglect the position dependent effects in the mass matrix. The incremental load rotation matrix, $[K_R]$, is nonsymmetric and causes some problems in applying Equation (3-6). Its effect when small increments are used is felt to be minimal and is ignored in SEADYN. The incremental damping matrix may be difficult to obtain in the more general situations. A simplified treatment is discussed in Section 3.3.

An alternative form of the incremental equations is obtained from Equation (3-1) by expanding only the internal loads in a Taylor series and neglecting higher order terms. The result is

$$ {}^{t+\Delta t}_{R}[M]\ {}^{t+\Delta t}_{R}\{q\} + {}^{t}_{R}[K_T]\ \{\Delta q\} = {}^{t+\Delta t}\{f\} - {}^{t}\{g\} $$

This equation is linearized by approximating ${}^{t+t\Delta t}\{f\}$ and ${}^{t+\Delta t}_{R}[M]$ with their values at t+Δt while remaining in the orientation defined by ${}^{t}C$. Any damping effects are assumed to included in ${}^{t+\Delta t}\{f\}$.

The contributions from each of the elements in the system (cables, lumped bodies, and rigid bodies) can be combined in a very simple and direct manner once they have been generated and transformed to the global coordinate system. This is done element by element by accumlating the element contributions in the appropriate position of the global arrays. An ordering of the degrees of freedom is implied in this procedure. The order assumed in SEADYN is simply the three displacement components (x,y,z) stored in the order of the node number. Thus, the global nodal displacement vector assumes the x component of node number one is first, the y component of node 2 is fourth, etc. By requiring slave (movement defined in terms of another node) nodes to be numbered after nodes which have active degrees of freedom, the solution bookkeeping is greatly simplified.

The assembled global equations have essentially the same form as the element motion equations. The main distinction is that the order of the equations is increased to include all of the active degrees of freedom in the system. Noting this, the total nonlinear equations of motion can be written

$$ {}^{t}_{R}[M]\ {}^{R}_{t}\{\ddot{q}\} = {}^{t}\{f\} - {}^{t}\{g\} = {}^{t}\{R\} \qquad \text{3-8} $$

The two incremental forms are

$$ {}^{t}_{R}[M]\ \{\Delta\ddot{q}\} + {}^{t}_{R}[C]\ \{\Delta\dot{q}\} + {}^{t}_{R}[\bar{K}_T]\ \{\Delta q\} = \{\Delta\bar{f}\} \qquad \text{3-9} $$

$$ {}^{t+\Delta t}_{R}[M]\ {}^{t+\Delta t}_{R}\{\ddot{q}\} + {}^{t}_{R}[K_T]\ \{\Delta q\} = {}^{t+\Delta t}\{f\} - {}^{t}\{g\} \qquad \text{3-10} $$

It should be emphasized that these represent the assembled equations for the system and that it is assumed that the constraints implied by the boundary conditions and slave/master conditions are accounted for. The dynamic equations reduce to the static equations when the time dependent terms are dropped. Thus, the nonlinear static equation is

$$ {}^{t}\{R\} = 0 \qquad \text{3-11} $$

and the incremental static equations are

$$ {}^{t}_{R}[\bar{K}_T]\ \{\Delta q\} = \{\Delta\bar{f}\} \qquad \text{3-12} $$

$$ {}^{t}_{R}[K_T]\ \{\Delta q\} = {}^{t+\Delta t}\{f\} - {}^{t}\{g\} \qquad \text{3-13} $$

In the static case the parameter t is used to signify a load step rather than a time step. The static equations presume a stable physical system has been described by imposing adequate constraints on the system. If this is not so, the stiffness matrices are singular and there is not a unique configuration of the system which will satisfy the equation. In case of nonlinear systems (particularly those with surface ships and mooring buoys) the static global equations may be ill-conditioned. This means they are nearly singular and numerical errors in the solution procedure may lead to apparent singularities. This will be given more attention in Section 4.2.

The classical approach for analyzing wave induced motions of platforms and vessels is to transform the incremental equations into the frequency domain and use linear superposition techniques. The linearized small displacement equation form used is that of Equation (3-9). The transforming assumption is that

$$ \{\Delta\bar{f}\} = R_e\left(\{F\}e^{i\omega t}\right) \qquad \text{3-14} $$

Assuming quasi-linearity of Equation (3-9), the steady state response has the form

$$ \{\Delta q\} = R_e\left(\{Q\}e^{i\omega t}\right) \qquad \text{3-15} $$

Substituting (3-14) and (3-15) into (3-9) yields

$$\left(-\omega^2 \, {}^t_R[M] + i\omega \, {}^t_R[C] + {}^t_R[K]\right) \{Q\} = \{F\} \qquad 3\text{-}16$$

Solution of this set of complex simultaneous linear equations allows the computation of the response amplitudes and phase angles for all degrees of freedom in the system. The magnitude of the response in each degree of freedom is given by

$$\left|Q_i\right| = Q_i Q_i^* \qquad 3\text{-}17$$

where Q_i^* is the complex conjugate of the i^{th} component of $\{Q\}$. The phase angle between the incident loading (wave) and the response is given by

$$\phi_i = \tan^{-1} \frac{I_m(Q_i)}{R_e(Q_i)} \qquad 3\text{-}18$$

A phase angle of zero corresponds to the response being in phase with the incident loading.

3.2 Equations for Line Elements

Two line element models are available. The first is the one-dimensional simplex element. This is a straight element using two nodes. The second element also uses two nodes in its definition and is in the form of a catenary.

3.2.1 The One Dimensional Simplex Element

A finite element which has the form of a line (i.e., one-dimensional) and uses only the field parameters at the two ends of the line in the interpolating function is referred to as a one-dimensional simplex element [11]. When the element is a straight line in 3D space and the field parameters are the nodal (end point) displacements the element is called a truss element in structural terminology.

Consider a single straight element which is defined by the position of two nodes (one at each end). Select a local coordinate system with the x axis extending from the first node to the second. The other two axes may be chosen arbitrarily under the restriction that they form a right handed cartesian reference frame. When the element is in its unloaded state it has a length ${}^oL$. Assume the material constitutive relation has the form

$${}^t_oS = {}^t_oE \, {}^t_o\varepsilon \qquad 3\text{-}19$$

where

^t_oS is the 2nd Piola-Kirchhoff stress in ^tC

$^t_o\varepsilon$ is the Green's strain in ^tC

^t_oE is a nonlinear material modulus which may be a function of strain.

The incremental form of this constitutive relation can be written for a small strain increment, $\Delta^t_o\varepsilon$.

$$^{t+\Delta t}_{\ \ \ o}S = {}^t_oS + {}^t_oE_T\ \Delta^t_o\varepsilon \qquad \text{3-20}$$

where $^t_oE_T$ is the tangent modulus evaluated at ^tC.

Making the finite element assumption that the displacement at any position along the element is a linear function of the displacements of the nodes one can write

$$\{u\} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \left[\left(1-\frac{x}{R_L}\right) I_3 \;\middle|\; \frac{x}{R_L}\ I_3 \right] \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \end{Bmatrix} \qquad \text{3-21}$$

$$= {}_R[N]\ \{q\}$$

where

$\{u\}$ represents the components of the displacement from ^RC

$\{q\}$ represents the components of the nodal displacements

$_R[N]$ is called a shape function matrix

I_3 is the identity matrix of order 3

The symbolic expression for the large displacement kinematic relations (Green's Strain) for a movement from ^RC to ^tC can be written [1]

$$^t_R\{\varepsilon\} = {}^t_R[D]\ {}^t_R\{u\} \qquad \text{3-22}$$

Substitution from Equation (3-21) yields

$$ {}^{t}_{R}\{\varepsilon\} = {}^{t}_{R}[D]\ {}_{R}[N]\ {}^{t}_{R}\{q\} = {}^{t}_{R}[B]\ {}^{t}_{R}\{q\} \qquad 3\text{-}23 $$

The mass matrix for the straight element can be written in two forms:

Consistent Mass Matrix

$$ [M] = \frac{{}^{R}\rho\ {}^{R}A\ {}^{R}L}{3} \begin{bmatrix} I_3 & \frac{1}{2} I_3 \\ \frac{1}{2} I_3 & I_3 \end{bmatrix} \qquad 3\text{-}24 $$

Lumped Mass Matrix

$$ [M] = \frac{{}^{R}\rho\ {}^{R}A\ {}^{R}L}{2} \begin{bmatrix} I_3 & 0 \\ 0 & I_3 \end{bmatrix} \qquad 3\text{-}25 $$

where ${}^{R}\rho$ is the element material density in ${}^{R}C$, and ${}^{R}C$, and ${}^{R}A$ is the element cross-sectional area in ${}^{R}C$. With the assumption of conversion of mass, the element mass matrix does not change with deformation. The pre-sub and superscripts are used in Equation (3-1) since this is not true of fluid added mass.

The consistent mass matrix is obtained from the kinetic energy and Equation (3-21). The lumped form can be obtained by the intuitive process of lumping half of the element mass at each node or by summing all the terms on each row of the consistent mass matrix and assigning the sum to the diagonal position.

The external forces may be due to point or distributed loads. Point loads appear as specific entries in the global equations. Distributed loads are usually from gravity effects and/or fluid loading. Fluid loading effects are discussed in Section 3.5. The general form of the gravity loading is

$$ {}^{t}\{f\} = \int_{0}^{{}^{R}L} {}^{R}[N]^{T}\ {}^{R}\{\Upsilon\}\ dx \qquad 3\text{-}26 $$

where

${}^{R}\{\Upsilon\}$ represents the components of the element specific weight (in fluid) relative to the local coordinate system

${}^{R}[N]^{T}$ is the transpose of ${}^{R}[N]$

Substitution from Equation (3-21) into Equation (3-26) and noting the orientation of the element with respect to the direction of gravity leads to the conclusion that these forces are equivalent to placing one half of the element weight acting in the gravity direction at each node. It should be noted that Equation (3-26) assumes mass is conserved.

The internal forces of Equation (3-1) can be written [1]

$$ {}^{t}\{g\} = {}^{t}_{R}S \; {}^{R}A \; \frac{{}^{t}L}{{}^{R}L} \begin{Bmatrix} -\lambda \\ \lambda \end{Bmatrix} \tag{3-27} $$

where $\{\lambda\}$ is a unit vector in the direction of the deformed element. The stress term, ${}^{t}_{R}S$, can be written

$$ {}^{t}_{R}S = \frac{{}^{t}P}{{}^{R}A} \frac{{}^{R}L}{{}^{t}L} $$

where ${}^{t}P$ is the element load in the deformed state. This allows the force residual to be written

$$ {}^{t}\{R\} = {}^{t}\{f\} - {}^{t}P \begin{Bmatrix} -\lambda \\ \lambda \end{Bmatrix} \tag{3-29} $$

The stiffness matrix can be obtained from consideration of the second variation of the strain energy in ${}^{t}C$. The result is [1]

$$ {}^{t}_{R}[K_T] = \int_{R}^{t} \left(\begin{bmatrix} k_o & -k_o \\ -k_o & k_o \end{bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} + \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} + \begin{bmatrix} k_G & -k_G \\ -k_G & k_G \end{bmatrix} \right) \tag{3-30} $$

where

$$ [k_o] = \frac{{}^{t}_{o}E_T \; {}^{o}A}{{}^{o}L} \left(\frac{{}^{R}L}{{}^{o}L} \right)^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} $$

$$ [k_1] = \frac{{}^{t}_{o}E_T \; {}^{o}A}{{}^{o}L} \left(\frac{{}^{R}L}{{}^{o}L} \right)^2 \begin{bmatrix} 2\theta_1 & \theta_2 & \theta_3 \\ \theta_2 & 0 & 0 \\ \theta_3 & 0 & 0 \end{bmatrix} $$

$$[k_2] = \frac{{}^{t}_{o}E_T \; {}^{o}A}{{}^{o}L} \left(\frac{R_L}{{}^{o}L}\right)^2 \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix}^T$$

$$[k_G] = \frac{{}^{t}P}{{}^{t}L} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\theta_1 = \frac{{}^{t}_{R}u_2 - {}^{t}_{R}u_1}{R_L}$$

$$\theta_2 = \frac{{}^{t}_{R}v_2 - {}^{t}_{R}v_1}{R_L}$$

$$\theta_3 = \frac{{}^{t}_{R}w_2 - {}^{t}_{R}w_1}{R_L}$$

3.2.2 The Bottom Limited Catenary Element

Quite often a mooring system will employ a line which must interact with the bottom as the moored body moves. The major feature of such a line is that significant lengths of line are lifted or laid down on the bottom when relatively small movements are induced at the upper end of the line. This is particularly true of shallow water moors. As an aid in modeling such lines SEADYN provides a bottom-limited catenary element.

The element uses the classical catenary relations with a modification to account for length changes due to line stretch. The nature of the catenary equations is such that it is not possible to solve explicitly for stiffness terms. A perturbation procedure presented by Peyrot and Goulois [12] is used to circumvent this problem. The SEADYN implementation follows the developments of Reference 12 with two exceptions. The first is that the tangential stiffness matrix produced by the perturbation procedure was recognized to be deficient since it did not recognize the geometric stiffness resisting out of plane motion. This deficiency was removed by adding a factor to the diagonal of the tangent stiffness matrix at the degree of freedom corresponding the out-of-plane displacement of each node. The factor added is the horizontal component of the catenary tension divided by the horizontal distance between the nodes.

The second modification consists of the imposition of a bottom limit on the catenary. The limit is assumed to be a horizontal plane passing through the lowest node on the catenary. The approach taken to enforce this limit is an iterative process similar to the one proposed by Peyrot [13].

The actual equations used are those of Reference 12 and will not be repeated here. The procedure relys on an algorithm which computes the nodal forces on a sagged line when the unstretched length, nodal positions and the distributed loading given. The tangent stiffness matrix is obtained from the changes in nodal loads produced by perturbing the nodal positions. Only cable weight loading is treated. Fluid drag loads are ignored on this element.

The bottom-limited catenary element uses a local coordinate system which has its origin at the bottom node. The local x axis is horizontal in the vertical plane containing the two element nodes. The positive x direction is toward the upper node. The local y axis is vertical upward. The z axis is chosen to form a right-handed cartesian system. Element stiffness and contributions to the residual are first computed in the local coordinate system and then transformed to the global system. The transformation procedures are those discussed in Section 3.9.

3.3 Material Models, Elasticity

The line material is assumed to be nonlinear elastic in form. Two functional forms are assumed which relate the line tension to the extensional strain. The first is a tabular form which represents a sequence of linear segments describing the relation. The second is a two parameter curve fit form. It has been found effective in modeling cable constructions [13]. The form is

$$t_p = a\,{}^t_o\varepsilon^b \qquad 3\text{-}31$$

where a and b are curve fitting parameters. Note that this relation is assumed to be between the total tension and the total uniaxial Green's strain. The relation between the customary uniaxial engineering strain and uniaxial Green's strain is

$$ {}^t_o\varepsilon = (1 + \frac{1}{2}\varepsilon_{engineering})\,\varepsilon_{engineering} \qquad 3\text{-}32$$

The difference between these two strain measures is minor up to strains of 10%. At an engineering strain of 20% the Green's strain is only 22%.

The material constitutive relations are used in two situations in SEADYN. The first occasion arises in the case where an initial equilibrium configuration is known (or guessed). In this case it is desired to find the unstrained length and the strain with the load and position of the nodes given.

In the other case, the unstretched length is known, the nodal positions are given and it is desired to compute the strains and the tangent modulus, ${}^t_oE_t$ °A. This situation occurs each time a new estimate of the state of the system is made.

In either case, it is necessary to use the expression for uniaxial Green's strain

$$ {}_{o}^{t}\varepsilon = \frac{1}{2}\left[\left(\frac{{}^{t}L}{{}^{o}L}\right)^{2} - 1\right] \qquad 3\text{-}33 $$

This form is used rather than accumulating strain increments to avoid numerical round-off errors. The ${}^{o}L$ for each element is either given or computed for each element at the beginning of the analysis. The ${}^{t}L$ is recomputed at each stage of the solution process where the strain is needed. The computation of these two lengths can be a source of numerical noise when ${}^{t}L \sim {}^{o}L$ and low precision is carried in the computer. Two situations aggravate the problem: (1) the material is very stiff, i.e., large ${}_{o}^{t}E_{t}$, (2) the loading is small.

3.4 Damping Models

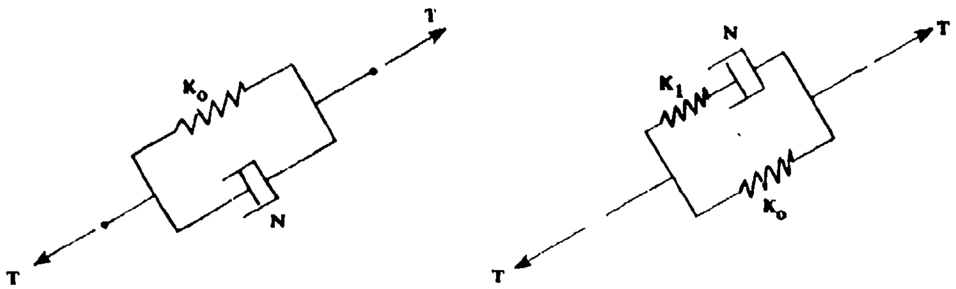
Damping enters the SEADYN analyses in various ways. First, there is a certain amount of damping inherent or intentionally included in the numerical algorithms for calculating transient response. Such algorithmic damping is treated in Section 5.2. Another form of numerical damping is involved in the nonlinear static solutions. This is an artifice to aid in obtaining convergent static solutions to highly nonlinear problems. Discussion of numerical damping can be found in Section 5.1. A third form of damping is due to the dissipative effects of the fluid interacting with the structure as discussed in Section 3.5.

This section is concerned with material or physical damping models for the dynamic response of the structure. Material damping is considered to be an inherent characteristic of the line material and is distributed throughout the line. It is appropriately treated in the material constitutive relation. The result is that the material takes a viscoelastic form. An alternative formulation presumes not enough information is available to define all features and sources of dissipation and the behavior is approximated through an estimated damping matrix. The form assumed is

$$ {}_{R}^{t}[C] = \alpha\, {}_{R}^{t}[M] + \beta\, {}_{R}^{t}[K] \qquad 3\text{-}34 $$

The treatment of material damping follows a component approach. Two components are used as building blocks: the elastic component which relates forces to displacements and the dashpot which relates forces to velocities.

These components may be used in two arrangements in SEADYN. These are referred to as the Kelvin and the NOAA-Reid models [25]. They are represented in Figure 3-1.



a) Kelvin Model   b) NOAA-Reid

Figure 3-1. Material Models

The force constitutive relations are given by:

Kelvin Model

$$T = T_K + T_N$$

$$T_K = K_o \delta$$

$$T_N = N \dot{\delta}$$

Hence 3-35

$$T = K_o \delta + N \dot{\delta}$$

NOAA-Reid Model

$$T = T_o + T_u \quad ; \quad \delta = \delta_o = \delta_u = \delta_1 + \delta_N$$

$$T_u = T_1 = T_N \quad ; \quad \dot{\delta} = \dot{\delta}_o = \dot{\delta}_y = \dot{\delta}_1 + \dot{\delta}_N$$

$$T_o = K_o \delta_o$$

$$T_1 = K_1 \delta_1$$

$$T_N = N \dot{\delta}_N$$

Hence 3-36

$$T = K_o \delta + N/K_1 (K_o + K_1) \dot{\delta} - N/K_1 \dot{T}$$

These relations can be written in terms of strains using the substitutions

$$K\delta = EA\varepsilon$$
$$N\dot{\delta} = \frac{CA}{L}\dot{\delta} = C A \dot{\varepsilon}$$

Then

<u>Kelvin Model</u>

$$T = EA_o \varepsilon + C A \dot{\varepsilon} \qquad 3\text{-}37$$

<u>NOAA - Reid Model</u>

$$T = EA_o \varepsilon + \frac{CA_1}{EA_1}(EA_o + EA_1)\dot{\varepsilon} - \frac{CA_1}{EA_1}\dot{T} \qquad 3\text{-}38$$

Equation (3-37) has been implemented in SEADYN by recognizing the EA_o term as the one discussed in Section 3.3. The CA term is taken as an additional input parameter.

The more complicated form of equation (3-38) presents more problems since it involves a tension rate term as well as a strain rate term. As with the Kelvin model, the EA_o term is identified as the basic material elasticity relation and the CA, and EA_1 parameters are provided as additional input. The tension rate is approximated by a backward difference in time. As a result it lags the other parameters. This approximation is considered to be a reasonable compromise.

These material damping models have only been implemented in SEADYN on the direct iterative (DI) method for transient dynamics (see Section 5.2). This solution works directly with the global equations in the form of Equation (3-8) and no global stiffness or damping matrix is used. The DI method makes the use of proportional damping a little less direct. In The case of proportional damping in transient dynamics using the DI method it is assumed that for each element

$${}^{t}T_N = (\alpha\, m\, {}^{o}L^2 + \beta\, {}^{t}_{o}E_S\, {}^{o}A)\, {}^{t}_{o}\dot{\varepsilon} \qquad 3\text{-}39$$

where ${}^{t}_{o}E_S$ is the material secant modulus.

The frequency domain solutions treat damping by forming an incremental damping matrix corresponding to the form of Equation (3-16). The material damping forms have not been implemented, but the proportional damping in the form of Equation (3-34) has. In addition, the dissipative terms from drag loading have been approximated using the approach presented in Appendix 1.

Neither material nor proportional damping is used in any transient dynamic option besides the D1 method. No damping is used in the natural frequency calculations.

3.5 Fluid Loads

The primary assumption regarding the effect of fluid immersion on the cable system as stated in Section 2 is that the fluid and structural problems are uncoupled. The independence principle [14] is assumed in the treatment of fluid loading on the cables. In brief, the independence principle asserts that the fluid loading can be treated as resulting from two separate flows: one normal and one tangential to the cable.

The fluid induced loads can be separated into a part involving the relative velocity between the cable and the fluid and a part involving the relative acceleration. The velocity related terms can be written

$$\{w\} = w_N \{\eta\} + w_T \{\bar{\lambda}\} \qquad \text{3-40}$$

where

$$w_N = 1/2 \, \rho_f \, C_N \, D \, V_N^2 \qquad \text{3-41}$$

$$w_T = 1/2 \, \rho_f \, C_T \, D \, V_T^2 \qquad \text{3-42}$$

$\{\eta\}$, $\{\bar{\lambda}\}$	are unit vectors in the directions of the normal and tangential components of the relative velocity
C_N, C_T	are normal and tangential drag coefficients
D	is the drag diameter of the cable
ρ_f	is the density of the fluid
V_N, V_T	are normal and tangential relative velocities

The fluid loading vector for an element is then

$$^t\{f\} = \int_0^{^tL} {^t[N]^T} \{w\} \, dL$$

Assuming a linear variation of $\{w\}$ over the length of the element

$$^t\{f\} = \frac{^tL}{6} \begin{bmatrix} 2\,I_3 & I_3 \\ I_3 & 2\,I_3 \end{bmatrix} \begin{Bmatrix} \{w\}_1 \\ \{w\}_2 \end{Bmatrix} \qquad \text{3-44}$$

The acceleration related portion of the fluid loading can be separated into a part due to flow field acceleration and a part due to structural acceleration. The flow field acceleration part is neglected in the cable loading and the structural acceleration part is treated as an added mass. The added mass matrix for a straight element can be written

$$[M_{added}] = \frac{C_M\, \rho_f\, {}^R\!A\, {}^R\!L}{2} \begin{bmatrix} 0 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \\ & & & & 1 \\ & & & & & 1 \end{bmatrix} \qquad 3\text{-}45$$

where

C_M is an added mass coefficient.

It should be noted that there is no added mass tangential to the cable which makes this a position dependent term.

In some of the solution procedures economies can be achieved if the mass matrix is diagonal. A lumped form of the combined cable and fluid added mass matrix can be written:

$$[M] = \frac{\bar{\rho}\, {}^R\!A\, {}^R\!L}{2} [I_6] - \frac{C_M\, \rho_f\, {}^R\!A\, {}^R\!L}{2} \begin{bmatrix} 1 & & & & & \\ & 0 & & & & \\ & & 0 & & & \\ & & & 1 & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix} \qquad 3\text{-}46$$

$$= [M_o] + [M_{NL}]$$

where

$$\bar{\rho} = {}^R\rho + C_M\, \rho_f \qquad 3\text{-}47$$

This form still presents some difficulties which will be dealt with in the discussion of the solution procedures.

Fluid loads on submerged lumped bodies are estimated using an approach similar to that used for the cable element. Two forms for lumped bodies are considered: spherical and an end-faired cylinder. The drag loading on a sphere is given by

$${}^t\{f_{sphere}\} = 1/2\ \rho_f\, C_D\, D\, V^2\ {}^t\{\lambda\} \qquad 3\text{-}48$$

where

D is the diameter of the sphere

C_D is the drag coefficient

V is the relative velocity between the fluid and the point where the sphere is located.

${}^t\{\lambda\}$ is a unit vector in the direction of relative velocity.

The added mass for a sphere is

$$[M_{added}] = \frac{C_M \rho_f \pi D^3}{6} [1_3] \qquad 3\text{-}49$$

The end faired cylinder loading and added mass is assumed to have the same form as that for a cable element except that the total effect is placed at a single point rather than being distributed between two nodes.

There are specific fluid effects peculiar to surface buoys and ships which are discussed in Sections 3.7 and 3.8.

3.6 Lumped Bodies

Two forms of lumped bodies are considered. The simplest form treats the body as a single point with three displacement degrees of freedom. The point is assumed to have mass but no rotational inertia. No elasticity effects are attributed to the body, but it may be a means of inducing fluid loads into the system (see Section 3.5). If the mass of the body is m, then the mass matrix is simply

$$[M]_{lumped} = m \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \qquad 3\text{-}50$$

This body affects only one node in the system.

The second form is that of a rigid body with spatial dimensions. The mass is still assumed to be concentrated at a single point but that point has six degrees of freedom. The mass matrix assumed for this case has specific forms only in the case of mooring buoys and surface ships. The program generates the mass matrix for mooring buoys and assumes the mass matrix is defined by input from the ship's motion file for surface ships. See Sections 3.7 and 3.8 for details. In either case the mass matrix is a 6 x 6 matrix.

3.7 Surface Ships and Platforms

The rigid body element is used to model ships and platforms (the term ship will be used to mean either one). A single node point is used to define the position of the ship. Since the node must express the angular as well as spatial position of the ship it is required to have six degrees of freedom. (The SEADYN program uses two consecutive nodes of three degrees of freedom each to define a ship.) Attachments of mooring lines and/or working lines are handled through the slave/master transformation.

In static analyses the steady state effects of winds and surface currents acting on the ship are treated as lateral and longitudinal forces and a yaw moment which are assumed to act at the ship's reference point. The values of these forces depend on the flow velocities and the angle between the ship's heading and the flow direction. Empirical load tables giving load coefficients versus heading or analytical load functions may be used. The empirical approach is given in NAVFAC DM-26 [15] and both approaches are summarized in the appendices of the user's manual.

The dynamic equations for the response of surface ships to waves are usually given in an incremental linearized form. These equations have the form

$$ {}_{t}^{t}[M_S + M_{AS}]\ \{\ddot{u}_S\} + {}_{t}^{t}[C_S]\ \{\dot{u}_S\} + {}_{t}^{t}[K_S]\ \{u_S\} = \{f_S\} \qquad 3\text{-}51 $$

where

$\{u_S\}$ represents the six components of ship's motion (surge, sway, heave, roll, pitch, yaw)

${}_{t}^{t}[M_S]$ is the ships mass matrix including rotational inertial terms

${}_{t}^{t}[M_{AS}]$ is the added mass due to fluid acceleration effects

${}_{t}^{t}[C_S]$ is an equivalent linerarized damping matrix

${}_{t}^{t}[K_S]$ is the ship's hydrostatic restoring matrix

$\{f_S\}$ are the point equivalent forces representing the wave induced exciting forces

In order to obtain this linearization it is usually assumed that the ship is driven by a simple harmonic wave. With this assumption, the forces, added mass and damping are frequency and heading dependent. In addition, the linearization of the roll damping term makes it dependent on the magnitude of the roll angle. Equations of the form of (3-51) can be obtained for slender bodies using strip theory [16]. A more general theory is required for other forms [17]. The SEADYN program assumes the values for these coefficients are provided through a data file. The format of that data file is described in Appendix A of the User's Manual and Section 5.3 of the Programmer's Reference Manual.

The element equation represented by Equation (3-51) can be manipulated as any other element equation and combined with the global equations of the system. No new concepts are involved in these operations.

3.8 Mooring Buoys

Ship's moors often involve surface buoys which support the mooring line and are connected to the ship through a hawser. This type of buoy usually remains on the surface where it is subjected to the effects of winds, currents and waves. The general form of the buoy motion equations linearized to represent small excursions from a static reference state can be written:

$$ {}^{t}_{t}[M_{SB}]\ \{\ddot{u}_{SB}\} + {}^{t}_{t}[C_{SB}]\ \{\dot{u}_{SB}\} + {}^{t}_{t}[K_{SB}]\ \{u_{SB}\} = \{f_{SB}\} \qquad 3\text{-}52 $$

where the various terms follow the previously established pattern. The forcing term represented by $\{f_{SB}\}$ deals only with wave excitation. Static load effects on a mooring buoy follow the same form as described previously in Section 3.6.

In order to avoid a great deal of complexity, it is assumed that the buoy is spherical in shape and that a local cartesian coordinate system is selected which is vertical in the z direction and has the incident wave traveling in the +x direction. No loss of generality is incurred with this choice of coordinate system on a spherical buoy since the character of the coefficients do not depend on the orientation or attitude of the buoy. The problem is further simplified if it is assumed that the buoy is homogeneous with the center of gravity at the geometric center of the buoy and that the geometric center is located at the water line in the reference state.

Attachments of hawser and mooring lines to the buoy can be readily handled if their positions relative to the local coordinate system are known. The rigid link transformation described in Section 3.11 is used for this purpose. The positions of the attachments can be found from the static solution.

Since it is assumed that the equations use ${}^{t}C$ as the reference configuration, the configuration notation will be dropped for this discussion.

Transformation from the local to the global system for assembly of the buoy equations with the rest of the moor system equations is a straightforward process which follows the method outlined previously. For this reason only the coefficients in the local coordinate system will be given here. It should be kept in mind that the following equations represent the incremental motion equations for a surface buoy in the local coordinate system just described.

Given that the buoy has a mass designated by m and a mass moment of inertia, J_m, the buoy portion of the mass matrix is

$$ [M_B] = \begin{bmatrix} m\,I_3 & 0 \\ 0 & J_m\,I_3 \end{bmatrix} \qquad 3\text{-}53 $$

where I_3 is the identity matrix of order 3. The assumption of a homogeneous sphere should be recalled at this point. If the attachments contribute significant mass, their effects can be treated by including additional lumped masses at those nodes.

The added mass has the form

$$[M_A] = \begin{bmatrix} A_{xx} & 0 & 0 & 0 & A_{x\theta} & 0 \\ & A_{yy} & 0 & A_{y\phi} & 0 & 0 \\ & & A_{zz} & 0 & 0 & 0 \\ & \text{(SYM)} & & A_{\phi\phi} & 0 & 0 \\ & & & & A_{\theta\theta} & 0 \\ & & & & & A_{\psi\psi} \end{bmatrix} \qquad 3\text{-}54$$

$$[M_{SB}] = [M_B] + [M_A] \qquad 3\text{-}55$$

The wave damping matrix has a form similar to the added mass matrix. Specific values for the added mass and damping coefficients for a sphere were given by Patton [20]. His values were obtained by curve fitting the analytical results presented by Kim [18]. The nondimensional values obtained were

$$M_x = 1.089 + 0.052\, a' \qquad \text{for } 0 < a' < 0.74 \qquad 3\text{-}56$$

$$= 1/(-0.0318 + 0.954\, a') \qquad \text{for } 0.74 < a' < 3.4$$

$$M_z = 1.85 \qquad \text{for } 0 < a' < 0.1 \qquad 3\text{-}57$$

$$= 1.02\, a'^{-0.256} \qquad 0.1 < a' < 3.4$$

$$N_x = 0 \qquad \text{for } 0 < a' < 0.1 \qquad 3\text{-}58$$

$$= -0.069 + 0.715\, a \qquad \text{for } 0.1 < a < 1.37$$

$$= 1.595 \qquad \text{for } 1.37 < a' < 3.4$$

$$N_z = 0.126 + 1.7\, a' \qquad \text{for } 0 < a' < 0.4 \qquad 3\text{-}59$$

$$= 1.18 e^{-0.83a'} \qquad \text{for } 0.4 < a' < 3.4$$

where a' is the nondimensional frequency given by

$$a' = \frac{2\pi a}{\lambda} = \frac{a\,\omega^2}{g} \qquad 3\text{-}60$$

and

a = radius of sphere

λ = wavelength

ω = circular frequency of wave container

These equations for the added mass and damping coefficients for mooring buoys proved to be cumbersome for programming. A polynomial curve-fitting of the original curves given by Kim [18] was used in the program. The polynomial coefficients are given below.

FUNCTION	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
M_x	1.0620	-0.4090	5.3299	-10.0143	7.6387	-2.9089	0.5509	-0.0414
M_z	1.7945	1.3362	-10.9227	18.0521	-13.8920	5.5725	-1.1249	0.0902
N_x	0.	-1.5252	7.2144	-8.6447	5.0535	-1.6218	0.2750	-0.0192
N_z	0.	4.3747	-9.8378	10.7232	-6.6657	2.3676	-0.4439	0.0339

The equation form is

$$F = \sum_{i=0}^{7} a_i (a')^i \qquad 3\text{-}61$$

The added mass terms are nondimensionalized by the factor ρa^3 and the damping terms by the factor $\rho a^3 \omega$, thus

$$A_{xx} = A_{yy} = \rho a^3 M_x \qquad 3\text{-}62$$

$$A_{zz} = \rho a^3 M_2$$

$$C_{xx} = C_{yy} = \rho a^3 \omega N_x \qquad 3\text{-}63$$

$$C_{zz} = \rho a^3 \omega N_2$$

The roll, pitch and yaw added mass terms arise from fluid viscosity and they can be written

$$A_{\phi\phi} = A_{\theta\theta} = A_{\psi\psi} = \frac{4\pi \rho a^5}{3} \frac{1 + \beta a}{1 + 2\beta a + 2\beta^2 a^2} \qquad 3\text{-}64$$

where

$$\beta = \sqrt{\frac{\omega}{2\nu}} \qquad 3\text{-}65$$

and ν = the kinematic viscosity of the fluid. Since the kinematic viscosity of water is of the order of 10^{-5} ft^2/sec, the rotational added mass terms will be small compared to the buoy inertia terms. Damping due to rotational motion is very small and will be neglected. Thus,

$$C_{\phi\phi} = C_{\theta\theta} = C_{\psi\psi} = 0 \qquad 3\text{-}66$$

When the center of pressure does not coincide with the center of gravity of the buoy, a coupling between lateral and rotational motion exists. These terms can be written:

$$A_{x\theta} = A_{xx}\,(z_{cg} - z_{cp}) \qquad 3\text{-}67$$

$$A_{y\phi} = -A_{yy}\,(z_{cg} - z_{cp})$$

$$C_{x\theta} = C_{xx}\,(z_{cg} - z_{cp}) \qquad 3\text{-}68$$

$$C_{y\phi} = -C_{yy}\,(z_{cg} - z_{cp})$$

With the origin of the local coordinate system at the geometric center (also center of gravity), z_{cg} is zero. The center of pressure for a half submerged sphere is obtained from

$$z_{cp} = \frac{\int_S z\,\eta_x\,dS}{\int_S \eta_x\,dS} = \frac{\int_0^{-a} z\sqrt{a^2 - z^2}\,dz}{\int_0^{-a}\sqrt{a^2 - z^2}\,dz} = \frac{4a}{3\pi} \qquad 3\text{-}69$$

The damping terms presented above do not represent the effects of viscous drag. The viscous terms involve the square of the relative velocity between the buoy and the fluid and are therefore nonlinear. The viscous effects are generally of less importance than the wave damping. Obviously, this is not the case for the rotational movement since those terms are zero for wave damping. In a free-floating buoy the viscous rotational terms would play an important part, but in a mooring system where the hawser and mooring leg restrain the buoy the rotation is limited. Therefore, all of the viscous terms will be neglected rather than attempting to linearize them.

The only nonzero hydrostatic restoring force on a half-submerged spherical buoy acts in the heave direction. Its value for small displacements is

$$k_{22} = \pi a^2 \rho g \qquad 3\text{-}70$$

When the buoy provides a connection between a mooring line and a hawser it develops an additional stiffness (resistance to motion) due to the tensile force being transmitted across it. This is analogous to the geometric stiffness term, $[K_G]$, seen in the cable element stiffness matrix in Section 3.2.1. This geometric stiffness effect is automatically taken into account by the rigid link transformation of the elements representing the attached lines.

The right-hand side of Equation (3-52) represents the forces due to surface waves. Assuming the wave is harmonic in form, Kim [19] shows that the wave induced forces can be written

$$\{f_{SB}\} = \left\{ \begin{array}{l} A^S_{xx}\,\ddot{X}_w + C^S_{xx}\,\dot{X}_w \\ \qquad\quad 0 \\ A^S_{zz}\,\ddot{Z}_w + C^S_{zz}\,\dot{Z}_w + I_1\,Z_w \\ \qquad\quad 0 \\ A^S_{\theta\theta}\,\ddot{\theta}_w + C^S_{\theta\theta}\dot{\theta}_w + I_2\theta_w \\ \qquad\quad 0 \end{array} \right\} \qquad 3\text{-}71$$

where

$$A^S_{xx} = \rho a^3 M^S_x \qquad 3\text{-}72$$

$$C^S_{xx} = \rho a^3\,\omega\,N^S_x \qquad 3\text{-}73$$

$$I_1 = \rho g \int_S e^{a'(z+ix)}\,\eta_z\,ds \sim \rho g \int_S \cos(a'x)\,\eta_z\,ds \qquad 3\text{-}74$$

$$I_2 = \rho g \int_S e^{a'(z+ix)}\,(x\eta_z - z\eta_x)\,ds \qquad 3\text{-}75$$

For the half submerged sphere

$$A^S_{\theta\theta} = {}^S_{\theta\theta} = I_2 = 0 \qquad 3\text{-}76$$

The wave pressure component of the heave exciting force, I_1, is closely approximated by the pressure at the water surface distributed over the cross section at the water surface. A plot of this function versus the nondimensional frequency is given in Figure 3-1.

The values for M^S_x, M^S_z, N^S_x and N^S_z for a half submerged sphere were by Kim (Ref 19). A polynomial curve fit of those functions plus the curve for I_1 are summarized in the following table.

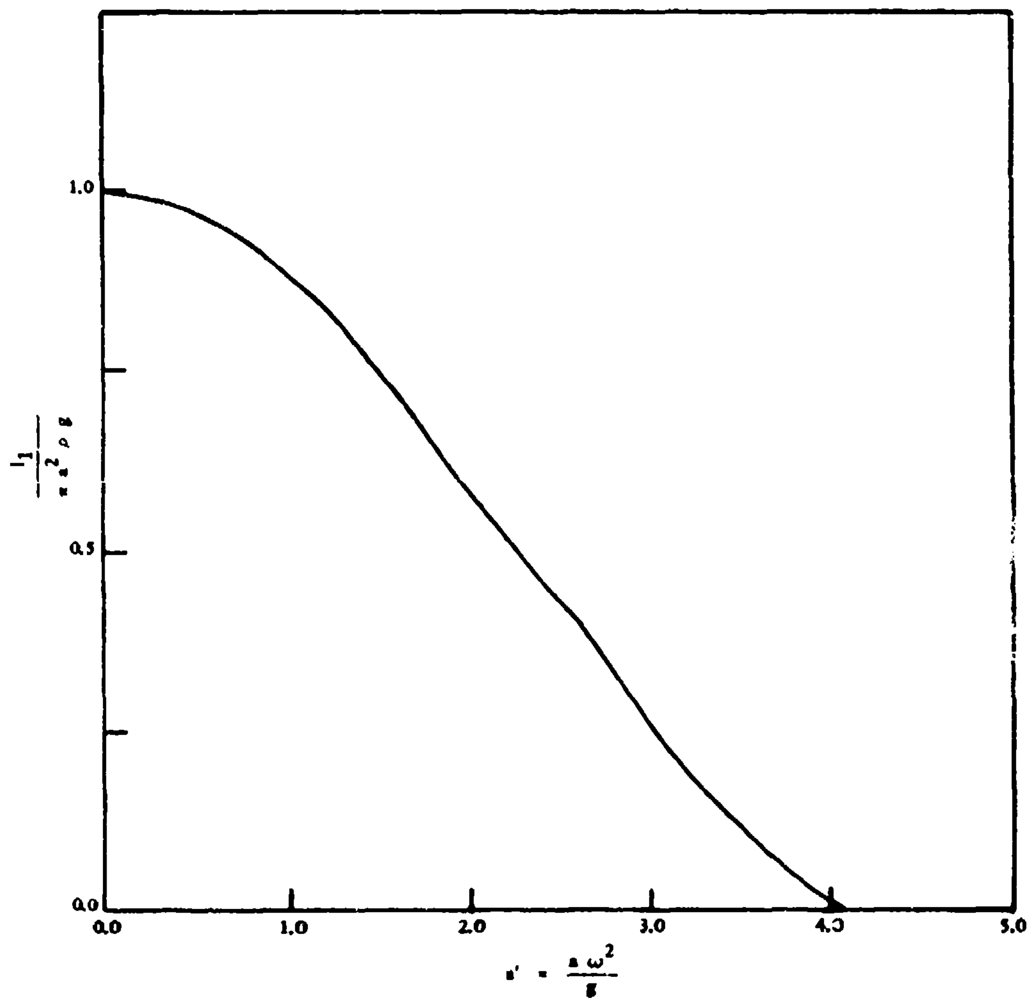


Figure 3-1. Heave exciting force for half-submerged sphere.

SUMMARY OF WAVE EXCITING FORCE COEFFICIENTS

FUNCTION	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
M_x^S	0.	1.7586	-8.2171	12.0253	-8.2882	3.0081	-0.5576	0.0416
M_z^S	1.7868	1.0552	-10.5792	17.2729	-13.2596	5.3272	-1.0781	0.0866
N_x^S	1.0833	0.0833	-1.4496	0.7753	-0.0705	-0.0314	0.0055	0.
N_z^S	0.	4.2382	-8.5367	8.2743	-4.6849	1.5633	-0.2823	0.0211
$I_1/\pi a^2 \rho g$	1.0	-0.0004	-0.1218	-0.0026	0.0069	-0.0007	0.	0.

where the function form is:

$$F = \sum_{i=0}^{N} a_i (a')^i \qquad 3\text{-}77$$

Thus all of the terms necessary for treating the small displacement behavior of a restrained, half-submerged spherical buoy are available. The assumptions employed appear to be reasonable and should lead to a good approximation of the buoy effects in a deep sea moor. Although a spherical buoy has been assumed, a comparison of the curves presented by Kim (Ref 18) for a sphere and those presented by Garrison (Ref 17) for a half-submerged cylindrical buoy with an aspect ratio of 1.0 shows that the added mass and damping coefficients are quite similar. Therefore, it is reasonable to expect the sphere equations to give at least an order-of-magnitude approximation of a cylindrical buoy.

The small displacement assumption deserves some further comment. For wavelengths of the order of the buoy diameter one would expect the buoy motion to be small. However, as the wavelength increases, the buoy motions increase. Since most of the wave energy is expected in the longer wavelengths, the buoy could be expected to see large motions which would cause these equations to be inaccurate. When no ship is in the system, this inaccuracy could be serious. Fortunately, the ship motion becomes a significant effect in the longer wavelengths and the buoy motion is dominated by the ship movement as transmitted through the hawser and reacted by the mooring line. In this case the contributions from the buoy itself (though in error) would generally be insignificant. For the shorter wavelengths, the ship appears nearly fixed, and the exciting forces due to wave action on the buoy become important. It is fortuitous that this is the range in which the buoy equations are most accurate.

3.9 Coordinate Systems and Transformations

Equations for the cable element, as well as ships and mooring buoys, are most readily developed in a local or intrinsic coordinte system which is considered to move with the element. The development of a global set of equations which represents the behavior of the assembled system of elements requires a single global coordinate system.

Transformations between the local coordinate system and the global system in a reference configuration are accomplished by the usual rotation of coordinates. The general form for the components of a vector at a point is

$$ {}^{t}_{R}\{u\}_{local} = {}_{R}[\hat{T}] \; {}^{t}_{R}\{u\}_{global} \tag{3-78} $$

Columns of the transformation matrix are the components of a unit vector in the direction of the local coordinate axis expressed in the global system. Since only right-handed cartesian coordinates are considered, the inverse transformation is obtained with the transpose of ${}_{R}[T]$. All contributions from the external and internal loads, etc., must be transformed to the global system before they are combined with other components in the system.

It should be noted that these coordinate transformations apply only to quantities expressed relative to a specific reference configuration. As long as the reference configuration remains fixed the individual coordinate transformations remain unchanged regardless of how much deformation occurs between ${}^{R}C$ and ${}^{t}C$.

Coordinate transformations involving nodal point displacements for an element can be written

$$ {}^{t}_{R}\{q\}_{local} = {}_{R}\left[\begin{array}{c|c} \hat{T} & \\ \hline & \hat{T} \end{array}\right] {}^{t}_{R}\{q\}_{global} = {}_{R}[T] \; {}^{t}_{R}\{q\}_{global} \tag{3-79} $$

The form for nodal forces is similar. When one transforms the incremental motion equations from the local to the global system the equations take the form

$$ {}^{t}_{R}[\bar{M}] \; \{\Delta\ddot{q}\}_{global} + {}^{t}_{R}[\bar{C}] \; \{\Delta\dot{q}\}_{global} $$

$$ + \; {}^{t}_{R}[\bar{K}_T] \; \{\Delta q\}_{global} = \{\Delta f\}_{global} \tag{3-80} $$

where

$$ {}^{t}_{R}[\bar{M}] = {}_{R}[T]^{T} \; {}^{t}_{R}[M] \; {}_{R}[T] $$

$$ {}^{t}_{R}[\bar{C}] = {}_{R}[T]^{T} \; {}^{t}_{R}[C] \; {}_{R}[T] $$

$$ {}^{t}_{R}[\bar{K}_T] = {}_{R}[T]^{T} \; {}^{t}_{R}[K_T] \; {}_{R}[T] $$

$$ \{\Delta f\}_{global} = {}_{R}[T]^{T} \; \{\Delta f\}_{local} $$

Thus, it is seen that coordinate rotations do not alter the form of the equations. Unless specific emphasis is required, no further distinction between the local and global equation forms will be made. It is assumed that the equations are written in a homogeneous system, i.e., all displacements, forces, stiffnesses, etc., are in the same coordinate system.

3.10 Restraint Transformations

A generalization of the coordinate system transformation is useful in modeling the effects of displacement restraints. Such restraints represent boundary conditions where the value of the displacement is specified. It may be zero or some finite quantity. SEADYN imposes these restraints in static analyses after the global stiffness matrix has been assembled. The process amounts to an imposition of a linear relation of the form:

$$\left\{\begin{array}{c} u_a \\ \hline u_d \end{array}\right\} = \left[\begin{array}{c} I_{aa} \\ \hline 0 \end{array}\right] \{u_a\} + \left\{\begin{array}{c} 0 \\ \hline d \end{array}\right\} \qquad \text{3-81}$$

Here it is assumed the [d] represents known or specified displacement components.

Consider a partitioned form of the static stiffness equations (the indices are dropped for simplicity)

$$\begin{bmatrix} K_{aa} & K_{ad} \\ K_{da} & K_{dd} \end{bmatrix} \begin{Bmatrix} u_a \\ u_d \end{Bmatrix} = \begin{Bmatrix} f_a \\ f_d \end{Bmatrix} \qquad \text{3-82}$$

Applying the linear transformation represented by (3-81) the following results

$$[K_{aa}] \{u_a\} = \{f_a\} - [K_{ad}] \{d\} \qquad \text{3-83}$$

The form of this equation suggests a procedure for imposing restraints on the global equations. First assume the d-portion has only one component, then take the column of the stiffness matrix that corresponds to that degree of freedom, and subtract it times d from the force vector. Next, set all entries in that row and column of the stiffness matrix to zero. Finally, replace the entry in the force vector for that row to the value of d and set the corresponding diagonal element of the stiffness matrix to unity. This process is repeated for each degree of freedom where a restraint is to be imposed. This process is valid for any value of d (including zero). When d represents a large movement it will be necessary to utilize some nonlinear solution algorithm to get an equilibrium solution. These methods are discussed in Section 5.1.

3.11 Slave/Master Transformations

The generalized rigid bodies used to model ships and mooring buoys require some special manipulations to connect them to the rest of the system. Their motions are assumed to be described by the six degrees of freedom of a single point. The attachments, however, will not connect to the rigid body at that node. Since the body is assumed to be rigid, it is possible to express the motion of any point on the body in terms of the motion of a single point and the relative positions on the body. The node used to model the body is called a master node. Any other point on the body is called a slave node. Given the six components of motion at the master node, the translation components of a slave node can be written:

$$\{q\}_{slave} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \Delta z & -\Delta y \\ 0 & 1 & 0 & -\Delta z & 0 & \Delta x \\ 0 & 0 & 1 & \Delta y & -\Delta x & 0 \end{array}\right] \begin{Bmatrix} \{q\} \\ --- \\ \{\theta\} \end{Bmatrix}_{master} \qquad \text{3-84}$$

or

$$\{q\}_{slave} = [\hat{T}_{SM}]\ \{q\}_{master}$$

where Δx, Δy, Δz are components of the distance between the two points measured from the master to the slave, i.e., $\Delta x = x_{slave} - x_{master}$, etc. The matrix $[\hat{T}_{SM}]$ can be viewed as a generalized form of the coordinate transformation represented by Equation (3-78) and the transformation procedures of the previous sections can be employed.

It should be noted that the slave/master transformation involves an alteration of the number of degrees of freedom. When an end of a cable element connects to a slave node, the application of Equation (3-84) in the transformation indicated by Equation (3-80) results in a stiffness matrix (etc.) which is 9 x 9 instead of 6 x 6. It should further be noted that the slave/master transformation form assumes small displacements and is therefore only applicable to the incremental equations.

$${}^{t}\{x\}_{slave} = {}^{t}\{x\}_{master} + {}^{t}_{R}[\hat{T}] \left({}^{R}\{x\}_{slave} - {}^{R}\{x\}_{master} \right) \qquad \text{3-85}$$

where ${}^{t}_{R}[\hat{T}]$ is a rotation matrix of the form (3-78) which represents the total angle changes from ${}^{R}C$ to ${}^{t}C$ at the master node.

4.0 MODELING CABLE AND MOORING SYSTEMS

4.1 General Approach

The SEADYN program utilizes a line element in such a way that it can represent truss structures as well as cables and mooring lines. All of these structures have a common feature in that the basic element is a one-force member. Modeling a cable span with multiple line segments or representing a truss structure involves essentially the same steps. Of course, a segmented model of a curved span represents some special approximations not needed for trusses and the geometric nonlinearities must be treated in cable spans while that may not be essential in trusses. But once the approximations have been made, the building block, the element, is one that can be used for either type of structure.

One should be aware that whenever only two line elements meet at a node and the other ends of those elements are not totally fixed, the geometric nonlinearity must be treated. Without preloading and appropriate external loads, such a system is a mechanism. The stiffness matrix for a mechanism is singular.

Although truss structures can be defined that have sufficient rigidity and structural stability that small deformation assumptions can give reasonable solutions, small deformation theory cannot be used on cable spans unless large preloads are supported and the stiffening effects of those preloads are included in the equilibrium equations.

In general, the modeling process for cable systems involves the selection of appropriate subdivision of the spans. Since straight lines are the predominant elements in SEADYN, it is necessary to make the subdivision fine enough to capture the curvatures of the initial and final states with acceptable error. More curvature requires more elements. Other modeling assumptions relate to material variations and lumped body approximations. Nodes must be located where materials change (only one material per element) and where bodies are to be lumped. Nodes also must be placed where limit conditions or other restraint/constraint conditions may be specified. Liberal use of nodes and elements has various economic impacts and good modeling practice leads to rational compromises between economics and accuracy.

Proper modeling procedures requires some special understanding of the loading environment and the possible boundary conditions. It is necessary to separate loading conditions into categories. Three categories dealt with in SEADYN are

DEAD - Loads inherent with the structure (weight, buoyancy) which have virtually no temporal variation. They must be supported along with all and any other loads.

LIVE - Loads which are temporary in nature but applied slowly enough to be assumed static. Current and wind loads can be placed in this category. Various operational or working loads are of this type.

Transient - Loads which excite or produce acceleration effects leading to movement in a time scale short enough to preclude static approximations. Generally this will mean oscillatory responses in which mass related forces are not negligible.

SEADYN assumes all three of these conditions will be encountered and since geometric nonlinearities are involved, the behavior in one condition is likely to be dependent on the state induced by other conditions. A typical situation is one in which dynamic responses are highly sensitive to the static state produced by a combination of gravity/boundary loads and current loading with negligible time variations but significant space variations. It is not appropriate to attempt dynamic analysis in this case until the static solution is found. The structure of SEADYN presumes a staged or sequenced analysis process which applies new loading conditions to the state obtained in the preceeding stage. Full account is given for the geometric and property changes in this staging. A typical analysis sequence would be to compute the equilibrium state for DEAD loads, apply the LIVE loads and finally induce the transient loads on the combined static state.

The primary variables in the computations are the nodal displacements and nodal velocities. Secondary information such as strains and tensions are computed from the primary data. Interpretation of results should always be tempered with an understanding of the approximations and methods used. Since the straight elements presume linear functions for displacements between the nodes only a constant value for element strain can be obtained. The consequence of this is a single value for the element tension.

Care is needed in specifying initial data for highly flexible structures with low preload. Such problems suffer from a tendency to radically change shape when small variations are made in element lengths, initial node positions or element tensions. One often encounters great difficulty in obtaining numerically stable initial configurations for dead loading. More is said of this in later sections and the various references.

4.2 Ship Mooring Systems

The basic components required in modeling mooring systems for surface ships are:

a. surface ship

b. mooring lines (usually submerged)

c. mooring buoys

d. hawser (usually in air and subjected to wind loads)

e. floats, sinkers and anchors

Each of these components have been dealt with to some extent in the previous discussion. Unfortunately it is not possible to develop a fully general nonlinear analysis of ship's mooring dynamics. The primary reason for this is the highly complex nature of the interaction between the sea and surface bodies such as ships and mooring buoys. The equations presented for these bodies are linearized equations which address only the response to harmonic, long-crested waves.

The theoretical approach used in dealing with mooring systems follows through a series of approximations. The first step is to obtain a description of the mooring system in the quiescent state where only gravity loads are involved. This is called a dead load analysis. There are some pitfalls in this step which are related to the geometric nonlinearity of the problem. One does not usually know, a priori, what the dead load configuration of the system is and the terms in the static equations are configuration dependent. This is the so-called initial configuration problem and it is dealt with in some detail in References 1 and 7. Section 4.8 also discusses the problem.

The active loads on the mooring system are assumed to be winds, surface and subsurface currents and surface waves. The effect of surface waves is primarily a dynamic phenomenon while the static or steady-state effects of winds and currents predominate. Therefore, it is assumed that the active loads on the system can be separated into a static effect, which is primarily due to winds and currents, and a dynamic effect from surface waves.

The next step in the analysis is then a static analysis to obtain the response to winds and currents. This is referred to as a live load analysis and it may include point loads representative of imposed work loads. Nonlinearities in the system also play an important part in a live load analysis. The geometric nonlinearity is still present. A significant new nonlinearity comes from the nonconservative, position dependent loads. The flow induced loads are strongly dependent on the orientation of the various elements and the loads change direction and magnitude as the system changes shape and position.

Material nonlinearities also play an important part at this stage. The most pronounced effect comes from the fact that the lines cannot support compressive loads. Should the imposed loads cause any of the legs to go slack (usually in taut moors with neutrally buoyant lines), the material stiffness goes to zero and the group of elements involved with the leg are part of an unstable structure and the stiffness matrix becomes singular.

An important feature in slack moors (negatively buoyant legs) is their ability to resist loads by changing shape and by lifting line or laying it down at the bottom. Modeling this interaction with the bottom using reasonably long cable elements produces some approximations that must be kept in mind while interpreting the results. An alternative is to use the bottom limited catenary element. This element also introduces approximations that must be considered.

In certain situations it is not possible (or feasible) to obtain a dead load configuration and then proceed to the live load analysis. Direct solution for the combined dead and live loads is usually required in these cases. One example is the solution for a single point moor. In this case the quiescent state is of little value and solution for the combined effects is usually quite easy to obtain if one has a good estimate of the total horizontal load to be supported by the moor. Another situation where this procedure may be required is in dealing with taut moors where legs go slack. In this case the slack legs could be ignored (not included in the model) and the dead load included in the live load analysis.

The analysis of wave induced dynamics begins with the static configuration developed by the wind and currents. Of necessity this is a frequency domain analysis. The equations presented for the ship and mooring buoy dynamics were obtained by assuming the excitation was from a harmonic wave. The linearization process renders the equations frequency dependent and limited to small motion amplitudes.

The harmonic loading input assumes a reference point which is the defining node for the ship. Phase angles on the loading are induced in the loading which are dependent on the harmonic wave length and the distance of the load point from the reference point. Response phase angles represent shifts from loading applied at the reference point.

The dynamic tension response in each element is obtained by adding the displacement increments to the nodal positions recalculating the element tensions from the constitutive relations, using the new lengths, and subtracting the tensions in the static reference state.

Wave loading produces mean and very low frequency forces which may have significant magnitudes on moored vessels. These depend on the responses and orientation of the vessel relative to the wave. These wave induced drift forces lead to slowly varying loads on the system which lead to shifts in the "static" reference position. The treatment of these forces requires at least two passes through the frequency domain solution. After the first, the magnitudes of the drift forces are estimated. Since these are predominately static loads, the static analysis should be repeated with the additional loading. If this results in significant movement, the frequency domain solution should be repeated on the new reference state.

4.3 Statics

Various load incrementing options are provided with the static analyses. In general, the gravity load will be increased in increments in a dead load analysis. Fluid induced loads are increased in increments during live load analyses. Both analyses allow point loads which are incremented from zero to the specified value, held constant, or reduced from the value to zero. It is usually assumed that gravity loads are held fixed during a live load analysis, but an option is provided which allows gravity load to be incremented during a live load analysis.

When ships are used in a static analysis the ship is constrained to remain on the surface. This normally means the ship is fixed in heave, roll and pitch. It is possible to input stiffness terms to allow small heave and angular motions out of the surface plane. No checks are made on the magnitude of those responses, so the user is warned to evaluate the validity of his results when these stiffnesses are used.

4.4 Dynamics, Time Vs. Frequency Domain

SEADYN provides computation options for dynamics in the time and frequency formats. Transient dynamic analysis in the time domain presumes all of the nonlinearities are active. It is further assumed that only the line elements provide equations for the elastic strain energy. Kinetic energy terms (mass related terms) are obtained from the lines and lumped bodies. Dissipative terms come from material damping and the drag loading. No equations are available within SEADYN for the rigid bodies. These typically require major computations using fluid/solid interaction equations which are beyond the scope of SEADYN.

A time domain solution is called for whenever the problem imposes motions/loads which result in large movements, load variations producing nonlinear material effects, compression of flexible lines (slack), and/or modifications of limit conditions. The time domain form is the only one capable of dealing with those nonlinearities. The restrictions mentioned above on rigid bodies eliminate a very important area of computation from transient analysis in SEADYN. On the positive side, SEADYN provides a significant capability for analysis of difficult dynamic problems: anchor deployment; array placement, adjustment and response; towing of lines and bodies in irregular paths; mooring leg response when attachment point motion is known, etc.

The frequency domain solution is a quasi-linear approach aimed directly at the moored ship/platform/buoy problem discussed in Section 4.2. It is applicable when the assumptions of DEAD, LIVE and wave loading are appropriate. It is significant in that it provides the ability to treat coupled responses between the mooring lines and the moored rigid bodies. Highly restricted models for spherical mooring buoys are built into SEADYN, but the body equations and loading functions must be provided externally for ship/platform components.

The time domain and frequency domain forms can be used in tandem to investigate mooring leg dynamics. The approach would be to follow the combined mooring solution through the frequency response to define the motion at the mooring line attachment. This motion is then converted to a time sequenced description which is used to drive the top of a statically preloaded mooring leg. This produces a more realistic definition of the vessel motion and then allows assessment of the mooring leg nonlinearities.

4.5 Natural Frequencies and Mode Shapes

SEADYN includes the option for computing natural frequencies and mode shapes simply for informational purposes. It provides data to no other option within SEADYN when the global structure is investigated. The computation of strumming induced drag amplification uses this option to get strum string mode shapes on subsets of the structure.

The computations for the global structure take the lumped (diagonal) mass matrix without correction for the lack of tangential added mass along with the tangent stiffness matrix for currently defined state. The tangential added mass correction is not taken since it leads to a non-diagonal mass matrix, and that requires additional storage. Computer storage is a serious issue on this option since the Jacobi method is used. The Jacobi method works entirely in core and requires the storage of a full nxn matrix.

4.6 Surface and Bottom Constraints

The foregoing discussion introduces the problem of constraining the model of the system to responses which lie between the natural boundaries imposed by the water surface and the bottom. One does not expect buoys or lines to rise out of the water, or lines and anchors to go below the bottom. The SEADYN program assumes the surface and bottom are flat and parallel. Checks are made at each step of static and time domain analyses to see if nodes of the system are within the imposed limits. To avoid the costly operation of checking all nodes in the system at every step, checking is only done on points where special limit conditions are specified. When one of the critical nodes is within a certain tolerance distance of the surface or bottom, it is constrained. For buoys or floats, this means that the node is held fixed in the vertical direction but free in the lateral directions. All three components, or only the vertical, may be fixed for anchors. Whenever the vertical resultant of all of the element tensions connecting to the point exceeds the sum of the external vertical loads at that point, then the constraint is released. The external loads are assumed to include the distributed loads from the elements and the weight or buoyant force from the lumped body.

When a solution step allows a critical node to move past the limit position, an overshoot procedure is activated. On incremental solutions this consists of a step division in which the portion of the original step which will satisfy the limit and the remainder of the step is done with the limit restraint imposed. The iterative solution imposes the limit by moving the over-shooting component to the limit state, setting flags to prevent immediate release of the constraint, and a continuation of the iteration.

4.7 Line Payout and Reel-in

The treatment of line payout and reel-in presents a host of problems not normally encountered in structural dynamics. The major problem is that the mass and elastic properties change with time. Not only does this produce direct changes in the structure, there are more subtle changes such as time step stability characteristics. For example, during reel-in an element may become sufficiently short to require reduction of the time step to obtain convergence. Other problems relating to geometry effects will be pointed out as this discussion proceeds.

The approach taken in SEADYN for payout/reel-in is based on incrementally changing an element's unstretched length. Since the load carried by an element is computed from element strain and element strain is a function of the ratio of stretched and unstretched lengths, the equilbrium state is unfluenced by this length change. It is also necessary to adjust the mass assigned to the nodes bounding the element. Except for some procedural bookkeeping details the process is quite simple. First, it is assumed the payout/reel-in occurs only at points where the motion is defined. It may be fixed or given specific movements. Second, the incremental change in element length is computed from the time step and the average payout velocity over that time. The node where the payout occurs is assigned that velocity acting in the instantaneous direction of the element. If the payout point is a moved point, the velocity of movement is added to the point. Mass adjustments are then made to the nodes bounding the element and the equilibrium iterations are pursued. At each stage of the iteration the velocity components at the payout point are recalculated to reflect line re-orientation.

One of the bookkeeping problems arises when payout accumulates large amounts of length in the element, or reel-in removes a major portion of the element. In either case, a process is initiated which modifies the number of elements and nodes in the system. During payout the modification involves a subdivision of the element and the addition of a new node. The process is called mitosis for obvious reasons. The subdivision uses linear interpolation to locate the new node a preselected distance from the payout point. That new nodal point is given the name (number) of the payout node and a new name (number) is assigned to the payout point to preserve the connectivity specifications. The subdivided element then has a part which no longer grows and a part which continues to grow.

The reverse process is still called mitosis even though that is a misnomer. In the reel-in process the element which shrinks to a specified length is removed from the system by adding its length and mass to the next element down the line and assigning the interfacing node to the reel-in point. The element thus removed is rendered inactive by giving it a zero unstretched length. The node removed is assigned a fixed status to avoid solution algorithm problems.

A further bookkeeping problem is encountered in mitosis when the next element to be added or the one to be removed is not of the same material as the one further down the line. This multi-material payout problem is dealt with by an approximation as outlined below. Consider a payout end with nodes and elements are inactive and awaiting payout as represented in Figure 4-1.

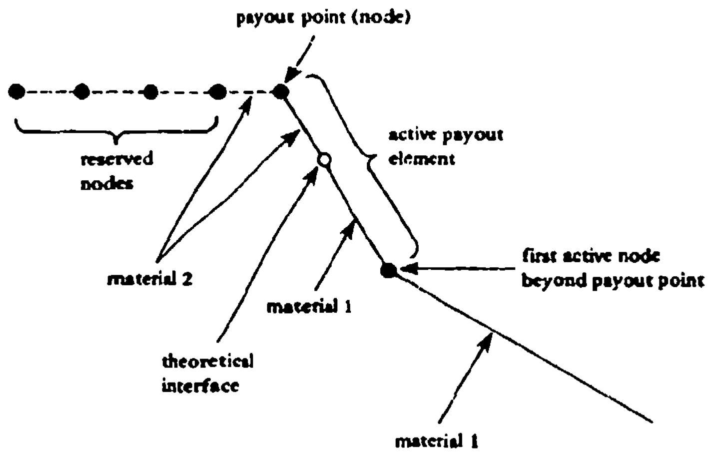


Figure 4-1. Multi-material Payout Configuration

At the beginning of the payout sequence the active payout element is entirely composed of material 1. The new length added to that element is material 2 since the next element to be made active is defined to be of material 2. Note that the reserved nodes are actually given the same coordinates with fixed conditions on all components and the elements connected to them are assigned an unstretched length of zero.

As the new length is added to the active element, two lengths are maintained for the element. The first is the length at the beginning of the sequence (i.e., the length of material 1) and the total length added (i.e., the length of material 2). Since there is no node at the theoretical interface the active element is treated as a composite element. The composite EA and effective tension are calculated as follows:

1. Use current nodal positions to calculate element stretch, ΔL.

2. Apportion ΔL: to ΔL_1, and ΔL_2 based on EA_1, and EA_2 from the preceeding time step or iteration.

3. Calculate T_1, T_2, EA_1, EA_2, from material properties using the new sub-lengths to calculate the strains.

4. Compare T_1 to T_2 and repeat steps 2 and 3 until T_1 is sufficiently close to T_2.

The formulas used to apportion ΔL to materials 1 and 2 are based on the small strain assumption as follows:

Given:

$${}^0L_1,\ {}^0L_2,\ {}^tL_1 + {}^tL_2,\ T,\ EA_1,\ EA_2$$

Find:

$$ {}^{t}_{0}\varepsilon_1,\ {}^{t}_{0}\varepsilon_2,\ \Delta L_1,\ \Delta L_2 $$

Known:

$$ {}^{t}T_1 = {}^{t}T_2 = {}^{t}T $$

$$ {}^{t}_{0}\varepsilon \sim \frac{\Delta L}{L} = \frac{{}^{t}L}{{}^{0}L} - 1 \qquad \text{small strain approximation} $$

$$ T \sim \frac{AE}{L}\,\Delta L $$

Hence:

$$ T = \frac{AE_1}{{}^{0}L_1}\left({}^{t}L_1 - {}^{0}L_1\right) = \frac{AE_2}{{}^{0}L_2}\left({}^{t}L_2 - {}^{0}L_2\right) \tag{4-1} $$

$$ {}^{t}L_1 = \frac{\dfrac{{}^{0}L_1}{AE_1}}{\dfrac{{}^{0}L_1}{AE_1} + \dfrac{{}^{0}L_2}{AE_2}}\left({}^{t}L - {}^{0}L\right) + {}^{0}L_1 \tag{4-2} $$

$$ \Delta L_i = \frac{\dfrac{{}^{0}L_1}{AE}}{\dfrac{{}^{0}L_1}{AE_1} + \dfrac{{}^{0}L_2}{AE_2}}\,\Delta L \tag{4-3} $$

The same equations are used in reel-in. The reel-in mitosis will assign the length of the removed element (material 2) to the next element down the line (material 1) by a ΔL equal to the length involved. The continued reel-in will then remove lengths of material 2 until the theoretical interface point is reached.

Yet another complication is encountered when lumped bodies are assigned to the nodes involved in payout/reel-in. When a lumped body is assigned to the payout node the theoretical interface is again employed. This is so even if no material change is indicated. Mass is then shifted from the payout point to the first active node as line is paid out. When payout mitosis occurs the mass is reassigned back to the payout node as it is moved out to the new active position. The reverse of this process is employed with reel-in of lumped bodies. The mass redistribution formulas are:

$$M_I = \frac{1/2\ {}^0L_2}{{}^0L} M_2 + 1/2 \frac{{}^0L_1}{{}^0L} M_1 + \frac{{}^0L_2}{{}^0L} M_B \qquad 4\text{-}4$$

$$M_0 = \frac{{}^0L_1 + 1/2\ {}^0L_2}{{}^0L} M_2 + 1/2 \frac{{}^0L_1}{{}^0L} M_1 + \frac{{}^0L_1}{{}^0L} M_B \qquad 4\text{-}5$$

$$\Delta M_I = \frac{M_2\Delta L}{2}\left[\frac{(2\,{}^0L_2 + \Delta L)\ {}^0L - {}^0L_2^{\,2}}{{}^0L\ ({}^0L + \Delta L)}\right] + \frac{M_1\Delta L}{2}\left[\frac{{}^0L_1^{\,2}}{{}^0L\ ({}^0L + \Delta L)}\right] \qquad 4\text{-}6$$

$$+ \frac{\Delta L\ {}^0L_1}{{}^0L\ ({}^0L + \Delta L)} M_B$$

$$\Delta M_0 = \frac{M\ \Delta L}{2}\left[\frac{(2\ L^0 + \Delta L)\ {}^0L - (2\ {}^0L\ + {}^0L\ )\ {}^0L}{{}^0L\ ({}^0L + \Delta L)}\right] \qquad 4\text{-}7$$

$$+ \frac{M_1\Delta L}{2}\left[\frac{-\ {}^0L_1^{\,2}}{{}^0L\ ({}^0L + \Delta L)}\right] + \Delta L\left[\frac{-{}^0L_1}{{}^0L\ ({}^0L + \Delta L)}\right] M_B$$

where

I refers to the first active node

0 refers to the payout node

$M_1 = {}^0L_1\ M_1$

$M_2 = {}^0L_2\ M_2$

m_1 = mass per unit of unstretched length, material 1

m_2 = mass per unit of unstretched length, material 2

M_B = mass of lumped body

A similar procedure is used to adjust the gravity loads.

The drag loads are apportioned as follows

$$F_{iI} = \frac{{}^0L_2}{{}^0L} F_i \qquad 4\text{-}8$$

$$F_{i0} = \frac{0_{L_i}}{0_L} F_i \qquad 4\text{-}9$$

Where F_i represents the components of the body dragload.

Finally, another bookkeeping feature has been found to be necessary when starting a payout sequence. Mitosis is defined to occur when the unstretched length of an element exceeds a user defined mitosis length plus a reference length. The reference length is the unstretched length of the element at its payout initiation or the mitosis length. The initial length is taken in those cases where there is multimaterials involved or where no previous mitosis has occurred on the line (i.e., at payout startup). The reference length is taken to be the mitosis length in all other situations. This multiple choice reference length is needed to avoid premature mitosis when starting with a long element, and to retain a consistent definition of the theoretical interface on multi-material payout.

4.8 Drag Amplification Due to Line Strumming

A problem of considerable importance in the analysis of underwater cable structures is the estimation of the forces induced by interactions of the structure with the moving fluid. The assumptions and drag models presented earlier neglected an important but ill-defined phenomenon. When the cable cross section is approximately round or when cable fairing has negligible torsional resistance, the steady-state value of the lift force is negligible. The foregoing developments have neglected this force. However, under appropriate combinations of flow conditions and cable size, a significant oscillating lift force can be produced by vortex shedding. If the frequency of this oscillatory lift force is sufficiently close to the frequency of a structural vibration mode, then significant structural motions are possible. Just how much motion results is a very difficult thing to predict.

There are many factors unfluencing the structural response of flexible systems to vortex shedding. Not all of them are understood. Undoubtedly such things as structural stiffness and orientation (both local and global), the proximity of abrupt changes in structural features, the coherence of vortex shedding, and the interaction between vortex formation and structure movement play some role in determining the response. A comprehensive theory which permits complete analysis is not yet available. Rather than wait for the advent of a complete theory (and in true engineering tradition), the analyst of cable structures introduces an approximation.

In many situations it is noticed that vortex shedding results in relatively small amplitude local responses. Rather than inducing direct changes in global structural response, the cable strumming action has indirect effects which are approximated quite well as an apparent increase in cable profile. This is in effect an increase in the normal drag coefficient. Capitalizing on these observations, Skop, Griffin and Ramberg [6] of NRL, formulated an approach to computing these drag amplifications.

Briefly stated, the NRL approach presumes a given segment of cable which is a candidate for strumming will strum when the vortex shedding frequency is in proximity of one of the cable natural frequencies. Specifically, strumming in the n^{th} cable mode is assumed to occur when

$$\omega_n \leq \omega_s < \omega_{n+1} \tag{4-10}$$

where

ω_n, ω_{n+1} are two adjacent structural natural frequencies.

ω_s is the vortex shedding frequency.

This presumes that the natural frequencies are closely spaced and tha no overlapping in responses occurs. When ω_s is below the lowest natural frequency, it is assumed that no strumming occurs. The same assumption is made when ω_s significantly exceeds the highest natural frequency.

Once the critical mode is identified a modal scaling factor is computed by

$$I_n = \frac{\int_0^L \psi_n^4 \, ds}{\int_0^L \psi_n^2 \, ds} \tag{4-11}$$

where ψ_n (s) represents the n^{th} mode shape.

The response amplitude along the string is estimated by

$$\frac{Y_{max}(s)}{d} = A_{max} \, I_n^{-1/2} \left| \psi_n(s) \right| \tag{4-12}$$

The value, A_{max}, is a dimensionless modal response parameter obtained from a least squares curve fit of experimental data relating it to a strumming response parameter, S_G.

$$A_{max} = 1.29/(1 + 0.43 \, S_G)^{3.35} \tag{4-13}$$

The parameter, S_G, is a function of the effective damping and the Vibratory Reynolds number. The latter is written

$$R_n = \frac{\omega_n d^2}{4\nu} \tag{4-14}$$

Where ν is the kinematic viscosity of the fluid and d is the effective cable diameter. The strumming response parameter is given by

$$S_G = 2\pi S^2 K_s \qquad 4\text{-}15$$

where S is the Strouhal number, 0.21, and

$$K_s = 22.2/R_n^{1/2} \qquad 4\text{-}16$$

The value of K_s reflects the neglect of internal cable damping and the assumption that the fluid damping coefficient per unit cable length is

$$C_f = 4.5\,\pi\,\rho\,\nu\,R_n^{1/2} \qquad 4\text{-}17$$

where ρ is the fluid density.

The strumming response estimate of Equation (4-16) is then used to calculate a wake response parameter as follows

$$W_r = (1 + 2\,Y_{max/d})\,(\omega/\omega_s) \qquad 4\text{-}18$$

Another curve fitting of experimental data leads to the following expression for the drag coefficient amplification

$$C_D(s)/C_{Do} = 1.0 \text{ for } W_r < 1.0$$

$$= 1.0 + 1.16\,(W_r - 1.0)^{0.65} \text{ for } W_r \geqq 1.0 \qquad 4\text{-}19$$

The integrated effect of the local drag amplification is then estimated by

$$C_{Dv}/C_{Do} = 1/L \int_0^L C_D(s)/C_{Do}\,ds \qquad 4\text{-}20$$

The procedure requires the following steps:

1. Identify a specific section of cable which is a candidate for strumming.

2. Develop an approximate local model for the cable and select a representative local fluid velocity.

3. Calculate ω_s

$$\omega_s = 2\pi S\,v/d\,\left|\text{Sin}\,(\theta_v - \theta)\right| \qquad 4\text{-}21$$

where

v = magnitude of local fluid velocity

θ_v = velocity heading

θ = representative cable heading

4. Calculate natural frequencies and mode shapes for the cable span.

5. Select critical mode.

6. If one is found, calculate drag amplification factor using Equations (4-11) through (4-20).

7. Assign this drag amplification to all elements of the string.

This procedure is repeated each time it is determined that significant changes have occurred in the flow or structural characteristics.

4.9 The Initial Configuration Problem

One of the frustrating features of cable analysis is that most cable systems do not have rigidity or spatial stability unless preloads are imposed. The systems are usually so flexible that small changes in preloads cause large shape changes. Noting that all of the static solution methods use a stiffness matrix, one is faced with a problem of getting a realistic estimate of the stiffness matrix which is nonsingular. In many situations it is necessary to have very accurate estimates of the initial configuration before any of the solution methods will work.

Various facets of this problem are explored in Reference 1 and 7. Some procedures which may help obtain stable starting configurations are discussed in the User's Manual.

A technique that has proven of some use in overcoming ill-conditioning and even singularities when using the MNR solution is the use of numerical damping. Felippa [21] shows that nonsingular adjustment to the estimator matrix, [K] can be generated by adding a matrix of the form

$$\mu \, B \, [I] \qquad 4\text{-}22$$

where

$$B = \{R\}^T [\tilde{K}] \, \{R\} / \{R\}^T \{R\} \qquad 4\text{-}23$$

and μ in a user specified numerical damping coefficient. This additional term tends to "stiffen" the estimator matrix and increase the chances for convergence.

The program also provides for numerical damping to be used with the incremental solutions. This feature should be used with extreme care since it alters the equations of equilibrium. If an appropriate value can be selected, it would be possible to compensate for some of the error in the first incremental step. There is no rational way available to estimate how much damping to use in this case.

The program provides a quick and convenient way of getting starting configurations when negatively buoyant lines are used. If a line between two defined points is a catenary which reaches the bottom with a horizontal tangent somewhere between the two points, then nodes can be generated along that line. The well known catenary equations are used and one or more of the generated nodes are assumed to be on the bottom if the lower defining node is not at the tangent point. The element preloads for the line are also generated and can be assigned to the line elements.

4.10 Component Adequacy Check

The SEADYN program provides a unique feature of checking the capacity of the various components of the system against imposed loads. The three types of checks provided are:

1. Anchors - the loads imposed on the anchor or fixed node are summed and the resultant is compared with the anchor holding power.

2. Buoys - the resultant of the loads in the lines connecting to the buoy is checked against the buoy capacity.

3. Lines - individual cable elements are checked to see if they are loaded beyond their capacities.

The component checks for buoys and anchors follow the procedures outlined in NAVFAC DM-26 and discussed in Reference 22. The component capacities can be input or obtained from the inventory tables developed for the DESMOOR program [22]. The inventories are described in Appendix D of the User's Manual.

All of the adequacy checks rely on estimates of the loads at the end of an element. The one dimensional simplex element has only one value of tension associated with it regardless of how long it is, what its weight is, and/or how much distributed load it is supporting. The tension associated with the element can be thought of as being at the midpoint of the element. The procedure for modeling distributed loads must be recalled to see how to estimate element end loads. Equations (3-26) and (3-43) both show that the nodal point equivalent loads for distributed loads are estimated using the element shape functions. These end loads represent the loads applied to the end nodes by the element as it supported distributed loads. The total load at each end of the element is then the vector sum of the element tension and the element loads. The direction of the resultant load at each end gives an estimate of the direction the cable lies at that point. In symbolic form the total loads applied to the nodes by the element are

$$
{}^{t}\{f\}_{elt} = \begin{Bmatrix} f_{x1} \\ f_{y1} \\ f_{z1} \\ f_{x2} \\ f_{y2} \\ f_{z2} \end{Bmatrix} = {}^{t}\{g\} + {}^{t}\{f\}_{grav} + {}^{t}\{f\}_{fluid} \qquad 4\text{-}24
$$

Some ambiguity in the loads applied to anchors results when long elements are used next to the anchor. If the line actually contacts the bottom before the anchor connection (i.e., between the two nodes which define the element), the SEADYN Program will not detect this in the solution. When a line check is made the resultant of the forces at the anchors will have a component pulling down indicating the line was hanging below the anchor. This is due to the model not being able to sense the bottom contact and transfer the weight at that point. Detailed and accurate modeling of the line interaction with the bottom requires short elements in that region. Fortunately the lack of modeling detail at the bottom of the line has little influence on the response at the top of the line.

5.0 SOLUTION METHODS

5.1 Static Solutions

The SEADYN program offers four basic solution methods for static analyses. The various features of each of them are discussed at length in References 1 and 7 so only a brief description will be given here.

The first method is a sequence of linear increments (SLI method). The loads are divided into a sequence of increments and the basic incremental Equation (3-12) is repeatedly applied. The SLI method requires the regeneration of the incremental stiffness matrix at each step to reflect the changes in position and constitutive relations. It has the undesirable feature of drifting from the correct solution through accumulating errors and small increments are required for accuracy.

The second method works with the incremental Equation (3-13). The first step is identical to the SLI first step, but on each succeeding step the force residual from the previous step is fed back as a corrector. For this reason it is called the residual feedback method (RFB). Although it is a non-iterative method it tends to be self-correcting. On responses which are reasonably well behaved (particularly monotonic responses) the RFB method gives accurate results with significantly fewer steps than does the SLI method. The RFB method costs somewhat more per step because of the residual calculation. The recalculation of the incremental stiffness matrix is still required at each step.

A more general method which employs iterations to solve Equation (3-11) is known as the modified Newton-Raphson method (MNR). It begins with an estimated configuration and uses an estimate of the tangent stiffness matrix to obtain successive displacement increments which hopefully lead to a zero force residual. Being an iterative method it is by far the most accurate of the three methods. This accuracy is not free, however. In the first place the method is not unconditionally convergent and the size of the load step required to assure convergence is not easily determined. In some cases it may be extremely small. In ill-conditioned systems it may not be possible to get convergence without some special auxiliary procedures.

The general form of the MNR method is

$$[\bar{K}]\ \{\Delta q\}^{(i+1)} = \{R\}^{(i)} \qquad \text{5-1}$$

$$\{q\}^{(i+1)} = \{q\}^{(i)} + \{\Delta q\}^{(i+1)}$$

where the superscript i refers to the iteration step. The $[\bar{K}]$ matrix is referred to as an estimator matrix. If $[\bar{K}]$ is the incremental matrix $[\bar{K}_T]$, and it is recalculated at each iteration, the usual Newton-Raphson procedure is obtained. If $[\bar{K}]$ is an approximation of $[\bar{K}_T]$ and/or it is not recalculated at each iteration one has a modified Newton-Raphson method. When $[\bar{K}]$ is not changed and the response curve is monotonically increasing function the successive estimates will usually oscillate about the correct solution. If the step size is small enough the estimates will tend towards the solution (converge). If the step size is too large the estimates will diverge.

Once oscillating estimates are detected, various accelerating procedures can be employed, and in some instances they will work even when the oscillations are divergent. The program detects oscillations by monitoring the degree of freedom with the largest initial response (i.e., largest component of $\{\Delta q\}^{(i)}$. Oscillation is signalled by repeated sign changes of the critical $\{\Delta q\}$ component. The simplest acceleration scheme averages the two alternating estimates using the sizes of the critical components to weight the average. An optional procedure uses a one-dimensional search to seek a close estimate of $\{\Delta q\}$ which crosses the correct solution. This search is initiated when the ith iteration signals a large oscillation. The search begins at the $(i-1)^{th}$ position and takes small steps in the direction of the i^{th} increment. A new step is tried at each position until the new increment reverses direction. At this point the last two positions are averaged to get a new starting estimate for the iteration. The size of the step used in this 1-D search is controlled by input. The input factor is the fraction of the i^{th} step which is taken as the first step of the search.

Various options are provided to measure convergence of the iterations. Reference 1 should be consulted for discussions of the convergence criteria. The options are listed in Sections 7.1.14 of the User's Manual.

The fourth solution method is called a viscous relaxation method. It is similar in many respects to the damped Newton-Raphson method. The major distinction is that the equations are cast into a transient dynamic format where a pseudo-time is used to control load steps and iterations. A pseudo-velocity vector is used in conjunction with the residual vector to make modifications in the damping and/or time steps. This allows the solution to be adapted to the character of the response. As damping becomes negligible, the method becomes the Newton-Raphson method. Details of the derivation are given in Reference 7. The solution form is

$$\left(\frac{1}{\alpha\Delta t}\,{}^{t}_{R}[C] = {}^{t}_{R}[K_T]\right)\{\Delta q\} = {}^{t+\Delta t}\{f\} - {}^{t}\{q\} + \left(\frac{1-\alpha}{2}\right){}^{t}_{R}[C]\;{}^{t}\{\dot{q}\} \qquad \text{5-2}$$

Approximating ${}^{t+\Delta t}\{f\}$ at the geometric positions of ${}^{t}C$ linearizes the method. It is applied the same way as the RFB method except that the time step, Δt, and the damping matrix are adjusted based on the process of the solution. The response data for the step are computed from

$$ {}^{t+\Delta t}\{q\} = {}^{t}\{q\} + \Delta t\;{}^{t}\{\dot{q}\} + \alpha\Delta t\;\;{}^{t+\Delta t}\{\dot{q}\} - {}^{t}\{\dot{q}\} \qquad \text{5-3}$$

$$ {}^{t+\Delta t}\{\dot{q}\} = \frac{1}{\alpha\Delta t}\left({}^{t+\Delta t}\{q\} - {}^{t}\{q\}\right) - \left(\frac{1-\alpha}{\alpha}\right) {}^{t}\{\dot{q}\} \qquad 5\text{-}4 $$

The damping matrix is computed from

$$ {}^{t}_{R}[C] = \left[{}^{t}_{R}C_{i}\right] \text{ (diagonal)} \qquad 5\text{-}5 $$

$$ {}^{t}_{R}C_{i} = {}^{t}\zeta(\bar{K} + \bar{C}_{i}) \qquad 5\text{-}6 $$

where

${}^{t}\zeta$ is a modifiable damping parameter

$\bar{C}_{j}$ is a response dependent term

$\bar{K}$ is a representative stiffness

The value of $\bar{K}$ is taken to be the average extensional stiffness (AE/L) or Equation (4-23). The $\bar{C}_{i}$ term is included for special problems associated with the angular responses of rigid bodies.

The pattern of velocity changes and the residual norm are used to adjust Δt and ${}^{t}\zeta$ in controlling the performance of the algorithm.

The form using Equation (5-2) is referred to as the VRR method. A form which avoids the residual calculation by replacing ${}^{t+\Delta t}\{f\} - {}^{t}\{g\}$ by $\{\Delta f\}$ is referred to as the VRS method. Its performance is inferior to the VRR method.

5.2 Time Domain Dynamic Solutions

The SEADYN program provides for four different time domain solution methods. Each of them utilize the following set of forward difference equations to develop the solutions:

$$ {}^{t+\Delta t}\{q\} = {}^{t}\{q\} + \Delta t\ {}^{t}\{\dot{q}\} + \frac{\Delta t^{2}}{2}\ {}^{t}\{\ddot{q}\} + \alpha\ \Delta t\ \left({}^{t+\Delta t}\{\dot{q}\} - {}^{t}\{\dot{q}\}\right) \qquad 5\text{-}7 $$

$$ + \beta\Delta t^{2}\ \left({}^{t+\Delta t}\{\ddot{q}\} - {}^{t}\{\ddot{q}\}\right) $$

$$ {}^{t+\Delta t}\{\dot{q}\} = {}^{t}\{\dot{q}\} + \Delta t\ {}^{t}\{\ddot{q}\} + \Upsilon\Delta t\ \left({}^{t+\Delta t}\{\ddot{q}\} - {}^{t}\{\ddot{q}\}\right) $$

where Δt is the time step and α, β, and Υ are integration parameters. The usual Newmark formulas are obtained with $\alpha = 0$.

When Equations (5-7) are augmented with a motion equation for the system, one has three simultaneous differential equations with the nodal displacements, velocities and accelerations at ${}^{t+\Delta t}C$ as unknowns. Specific forms of the solution routines for this set of equations are developed in Reference 1. Only the final forms will be presented here.

Direct Iteration Method (DIM)

This method uses Equations (5-7), (3-8), and (3-46) to obtain an iterative formulation. The form is

$$ {}^{t+\Delta t}\{q\}^{(n+1)} = {}^{t+\Delta t}\{q\}^{(h)} + (\alpha\Upsilon+\beta)\,\Delta t^2\,\Delta({}^{t+\Delta t}\{\Delta\ddot{q}\}^{(n+1)}) \qquad 5\text{-}8 $$

$$ {}^{t+\Delta t}\{\dot{q}\}^{(n+1)} = {}^{t+\Delta t}\{\dot{q}\}^{(n)} + \Upsilon\Delta t\,\Delta({}^{t+\Delta t}\{\Delta\ddot{q}\}^{(n+1)}) $$

$$ ({}^{t+\Delta t}\{\Delta\ddot{q}\}^{(n+2)}) = -{}^{t+\Delta t}\{\ddot{q}\}^{(n+1)} + [M\,]^{-1}\,({}^{t+\Delta t}\{R\,\}^{(n+1)} $$

$$ -\,{}^{t+\Delta t}[M_{NL}]^{(n+1)}\,{}^{t+\Delta t}\{\ddot{q}\}^{(n+1)}) $$

The iteration can be started with Equations (5-7) with $\alpha = \beta = \Upsilon = 0$ or with the residual feedback solution to be described below. The DIM method has been shown to be an accurate and cost effect approach. Since it is an iterative method which retains all of the nonlinear terms it readily deals with all of the important nonlinear effects such as position dependent loads, constraints, slack segments, and changing geometry. Of particular interest is the ease with which problems with defined motion are dealt with. When problems require the imposition of a known motion (e.g., cable towing problems) the moved node is simply held at the required dynamic state during the iterations.

Sequence of Linear Increments (SLI)

This method starts from Equations (5-7) and uses the incremental form (3-9). An iterative solution is also employed, but it has been shown that this method costs essentially the same as the DI method and is much less accurate [1]. It, too, is capable of solving the moving boundary problem but an augmented global stiffness matrix involving the moved degrees of freedom is required. This increases the storage requirements, reduces the speed and increases the numerical error potential.

Residual Feedback Method (RFB)

Equations (5-7) are inverted and substituted into the incremental form (3-10). The result is

$$ [K_{eff}\,]\,\{\Delta q\} = \{f_{eff}\} \qquad 5\text{-}9 $$

where

$$[K_{eff}] = [K_t] + \frac{1}{\Delta t^2 \beta} [M({}^t\{q\})] \qquad 5\text{-}10$$

$$\{f_{eff}\} = {}^{t+\Delta t}\{f({}^t\{q\})\} - {}^t\{g\} + [M({}^t\{q\})] \left(\frac{1}{\Delta t \beta} {}^t\{\dot{q}\}\right.$$

$$\left. - \left(1 - \frac{1}{2\beta}\right) {}^t\{\ddot{q}\}\right) \qquad 5\text{-}11$$

Equation (5-9) is a linear algebraic equation which can be solved by the usual manner. No iteration is required and the method has a self-correcting feature similar to the static RFB method. This method follows more closely the traditional Newmark implicit integration form [23]. It has the unconditional stability features that have made the method so popular. No provision is made for solving the moving boundary and payout problems, however. The fact that the RFB method requires the formation of a global stiffness matrix, the evaluation of a residual, and the solution of simultaneous equations reduces its cost effectiveness. This is usually compensated for by using larger time steps.

<u>Modified Newton-Raphson Form (MNR)</u>

This is an iterative method based on a procedure similar to the MNR static solution. The form is

$$[K_T]\, \Delta\left({}^{t+\Delta t}\{\Delta q\}^{(n+1)}\right) = {}^{t+\Delta t}\{R\}^{(n)} - {}^{t+\Delta t}[M]^{(n)}\, {}^{t+\Delta t}\{\ddot{q}\}^{(n)} \qquad 5\text{-}12$$

$${}^{t+\Delta t}\{q\}^{(n+)} = {}^{t+\Delta t}\{q\}^{(n)} + \Delta\left({}^{t+\Delta t}\{\Delta q\}^{(n+1)}\right)$$

$${}^{t+\Delta t}\{\dot{q}\}^{(n+1)} = {}^{t+\Delta t}\{\dot{q}\}^{(n)} + \frac{\Upsilon}{\Delta t\ (\alpha\Upsilon+\beta)} \Delta\left({}^{t+\Delta t}\{\Delta q\}^{(n+1)}\right)$$

$${}^{t+\Delta t}\{\ddot{q}\}^{(n+1)} = {}^{t+\Delta t}\{\ddot{q}\}^{(n)} + \frac{1}{\Delta t^2\ (\alpha\Upsilon+\beta)} \Delta\left({}^{t+\Delta t}\{\Delta q\}^{(n+1)}\right)$$

The iteration is started with

$${}^{t+\Delta t}\{q\}^{(0)} = {}^t\{q\} + {}^{t+\Delta t}\{\Delta q\}^{(0)} \qquad 5\text{-}13$$

$${}^{t+\Delta t}\{\dot{q}\}^{(0)} = \frac{\Upsilon}{\Delta t\ (\alpha\Upsilon+\beta)}\, {}^{t+\Delta t}\{\Delta q\}^{(0)} + \left(1 - \frac{\Upsilon}{(\alpha\Upsilon+\beta)}\right) {}^t\{\dot{q}\}$$

$$+ \frac{\Delta t(\alpha\beta-\Upsilon)}{2(\alpha\Upsilon+\beta)}\, {}^t\{\ddot{q}\}$$

$$ {}^{t+\Delta t}\{\ddot{q}\}^{(o)} = \frac{1}{\Delta t^2\ (\alpha\Upsilon+\beta)}\ {}^{t+\Delta t}\{\Delta q\}^{(0)} - \frac{1}{\Delta t\ (\alpha\Upsilon+\beta)}\ {}^{t}\{\dot{q}\} $$

$$ + \left(1 - \frac{(1+2\alpha)}{2(\alpha\Upsilon+\beta)}\right)\ {}^{t}\{\ddot{q}\} $$

where ${}^{t+\Delta t}\{\Delta q\}^{(0)}$ is obtained from an RFB solution. As an alternative to Equations (5-13) the iterations can be started from Equations (5-7) with $\alpha = \beta = \Upsilon = 0$.

The accuracy of this method has been demonstrated but its cost per time step does not appear competitive with the DI method [1]. Moving boundary and payout solutions have not been implemented with this solution.

It would appear from the studies done in Reference 1 that the best general purpose solution method of the four is the DI method. On some problems the method may encounter some difficulties, however. The iterations are convergent only for step sizes of the order it takes an axial wave to traverse the shortest, stiffest element. When the imposed forces are slowly varying the routine will attempt to increase the step size beyond stable behavior limits, because the motions are small in small time steps and the iterations converge rapidly. Controls are provided to the user to prevent step size increases in this situation.

In some situations the RFB solution may be found very cost effective. Generally this will be nearly linear systems, which are slowly varying. The user is cautioned against using very large steps with the RFB method since gross errors may result [1].

It should be noted that the diagonal form of the mass matrix, $[M_o]$, is used in each of the above methods. The advantages in storage and solution effort are obvious. It has been shown that with proper selection of the integration parameters, the errors introduced by the lumped mass approximation tend to compensate for those induced by the solution algorithm to give more accurate results. Experience has shown that $\alpha = 0$, $\beta = 1/12$ and $\Upsilon = 1/2$ is good choice of parameters. Preliminary observation [1] indicate that a small negative value for α may be beneficial in controlling amplitude attenuations. Values of $\Upsilon > 1/2$ introduce numerical damping of the higher frequency components while $\Upsilon < 1/2$ introduces negative damping.

5.3 Frequency Domain Solution

5.3.1 Solutions for Regular Waves

The response amplitudes, $\{Q\}$, for a given frequency are obtained by solving the linear simultaneous algebraic equations represented by (3-16). The coefficient matrices for mass and damping must be recalculated for each frequency since the linearization procedure leaves them dependent on the wave frequency.

The damping terms present some further difficulty. In addition to being frequency dependent, the linearized viscous terms are dependent on the amplitude of the response. The ship's roll-damping depends on the roll angle, the buoy rotational damping depends on the rotation angle, and the cable and lumped body damping depend on the lateral displacement amplitudes. Thus, it is seen that the incremental equations are not strictly linear.

An approximation procedure is introduced to deal with this problem. This involves iterative solutions of the Equation (3-16) for each frequency. The first solution at a given frequency is calculated assuming a ship's roll angle. The roll angle obtained from the solution is then used along with the other pertinent response amplitudes to recalculate the damping terms and obtain another solution. This procedure is repeated until two successive estimates of ship's roll are within 1° of each other. It is assumed that buoy and cable damping are less important than ship's roll and are thus converged when the roll has converged. The response is then dependent not only on the wave frequency, but also on the wave amplitude. This means that it is not appropriate to assume a unit amplitude for a given wave frequency with the intent of obtaining a Response Amplitude Operator (RAO) for that wave. It is necessary to have the correct wave amplitude at each frequency. Therefore, the sea spectrum must be used in the calculation of the regular wave responses. Once the steady state response for a given wave frequency and amplitude is obtained, the RAO is estimated by dividing by the wave amplitude.

The program allows the mass matrix to be formed either with the lumped or consistent form of the element mass. The lumped form is used for the fluid added mass terms.

Provisions for internal damping effects are provided in the proportional damping form. Thus

$$[C] = \alpha\,[M] + \beta\,[K_T] \qquad 5\text{-}14$$

where α and β are proportionally constants. The damping from fluid drag effects on the cable elements must be linearized before it can be used in Equation (3-16). This linearization is described in Appendix I.

5.3.2 The Steady-State Wave-Induced Drift Forces

Whenever waves encounter a floating body there arises a set of forces which tend to move the body in the direction the wave is traveling. These forces are often neglected since they are usually small in magnitude. These are the so-called second order wave-induced drift forces. They are generally slowly varying compared to the frequency of the incident wave; however, they have an average or steady-state component which may be significant enough to cause an adjustment of the static position of the ship. These forces are directionally dependent and are sensitive to the amplitude of the ship's response to the wave. The DTNSRDC Ship's Motion File provides a table of coefficients which can be used to estimate the steady-state drift forces after the ship's dynamic response is obtained. It should be noted that these forces do not estimate the dynamic effects, which are at a lower frequency than the incident wave.

The specific form for the drift forces is given in Appendix A of the User's Manual in the discussion of the Ship Motion File. The result is a set of forces for the lateral and longitudinal directions and a yaw moment acting at the ships reference point for each wave frequency. It is assumed that the drift forces are cumulative for the various wave represented in a wave spectrum. Therefore, the drift forces for each of the regular waves are accumulated. It is felt that this is a reasonable approximation since the wave amplitude indicated by the wave spectrum is used in response calculation.

These accumulated drift forces can then be used as an additional static loading to adjust the static reference state. If it is felt that this adjustment will affect the regular wave solutions significantly, the user can request iterations on the regular wave solutions and the drift force adjustments to the static reference until sufficiently small changes are found. Either the MNR, VR, or RFB solutions can be used to adjust the static reference.

5.3.3 Solution Procedure for Random Seas

The superposition of the regular wave responses to represent the response to random seas follows the well established methods from the theory of random vibrations [24]. The sea is assumed to be uni-directional (long-crested) and is described in terms of a generalized spectral energy density function having the following form:

$$S(\omega) = A/\omega^5 \, e^{-B/\omega^4} \qquad 5\text{-}15$$

Typical values for the parameters A and B are given in Section 7.3.2 of the User's Manual.

The frequencies to be included in the set of regular wave calculations are determined by specifying a frequency increment, $\Delta\omega$, a lower bound ω_{min}, and upper bound, ω_{max}. Regular wave responses are than calculated for

$$\omega_1 = \omega_{min} + \frac{1}{2}\Delta\omega + (i-1)\Delta\omega$$

$$\omega_i < \omega_{max}$$

At each frequency the incident wave height is determined from the sea spectrum by the following:

$$h(\omega_i) = (\tfrac{1}{2}S(\omega_i)\Delta\omega)^{1/2} \qquad 5\text{-}16$$

(Note that the wave height is twice the wave amplitude.) This wave height is then used to converge on a steady-state dynamic response using the methods described in the previous section.

Let $H(\omega_i)$ represent the response of one of the quantities (nodal displacement component or element tension). The response spectral density of this quantity is then given by

$$S_x(\omega) = HH^* \, S(\omega) \qquad 5\text{-}17$$

The mean square of the response is

$$E[x^2] \sim \int_0^\infty HH^* \, S(\omega) d\omega = \int_0^\infty S_x(\omega) d\omega \qquad 5\text{-}18$$

The integral is evaluated numerically using the points obtained from each of the regular wave solutions. Thus

$$E[x^2] \sim \sum_{i=1}^{N} S_x(\omega_i)\Delta\omega \qquad 5\text{-}19$$

where N is the number of regular wave components used.

When the response quantities represent the dynamic excursions relative to the static reference state, then their expected values (i.e., their means) are zero. The magnitude of the response is then treated as a static part plus a dynamic part. The amplitude of the dynamic part is assumed to follow a Rayleigh distribution with a mean-square value which is a function of the area under the response spectrum curve. Assuming the sea spectrum used is based on double the square of the wave height, the mean-square of the response amplitude is the value obtained (5-19) divided by eight. It is possible, then, to make statistical estimates of the maximum response by making statistical estimates of the dynamic part and adding them to the static values obtained in the updated reference configurations.

5.4 Natural Frequencies and Mode Shapes

The incremental Equations (3-9) without damping or external forces has the form

$$[M] \{\ddot{u}\} + [K_T] \{u\} = 0 \qquad 5\text{-}20$$

If it is assumed that simple harmonic motion occurs then

$$(-\omega^2 [M] + [K_T] \{u\} \sin \omega t = 0 \qquad 5\text{-}21$$

since $\{u\} \sin \omega t = 0$ only in the trivial case of $\{u\} = 0$, this requires that

$$\det ([K_T] - \omega^2 [M]) = 0 \qquad 5\text{-}22$$

Solutions to this equation lead to the natural frequencies and mode shapes of the small displacement free oscillations about the configurations used in expressing [M] and [K_T]. An iterative solution is used which obtains all of the natural frequencies and mode shapes for the system.

The primary assumption made in getting the frequencies and mode shapes is that the equations are linear and no significant damping exists. Both of these assumptions may be violated to some extent for underwater cable structures. The option is provided since the frequencies and mode shapes may still be indicative of the structural behavior.

5.5 Time Sequenced Static Solutions

Often the transient response problem involving boundary movement and/or line payout can require considerable computer resources to evaluate. In situations where imposed velocities and/or payout rates are relatively slow, approximate solutions can be obtained by a sequence of static "snap shots." SEADYN has implemented an option to drive the static solutions sequentially through boundary motion and payout conditions. This option is called the Time Sequenced Static Solutions (TSSS). Time is simply a parameter in this case which determines where the moving boundary is and how long the unstretched payout elements are. Only the iterative static solutions (MNR, VRR, VRS) are valid in this option. The previous state is used as a starting guess for each iteratively solved "snap shot." Time steps can be as large as the starting guess will allow; i.e., as long as the previous state remains a reasonable guess for the next state.

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